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Accounting for Growth Disparity: Lucas's Framework Revisited

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Abstract

This paper proposes a theoretical method to account for historical episodes of growth disparity, which are famously discussed by Lucas (1993). A numerical computation shows that the properties of the local dynamics of the proposed model are consistent with the facts indicated by selected episodes.

JEL classification numbers: I18; O11; O41

Keywords: Growth episodes; Public health infrastructure; Multiple equilibria; Indeterminacy; Saddle-path stability

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Highlights

- We revisit the famous growth episodes considered by Lucas (1993).
- Our dual-equilibrium model is an illuminating explanation for the selected episodes.
- In the model, high-growth equilibrium is locally indeterminate.
- In the model, low-growth equilibrium is locally saddle-path stable.

1. Introduction

A seminal paper by Lucas (1993) provided a framework to model income growth disparity during national economic development. The aim of the present study is to take that framework into consideration and address the problem from a new viewpoint based on a simple growth model with multiple equilibria. The growth episodes in the Philippines and South Korea considered by Lucas are frequently discussed in the literature on growth, but here we quote Lucas to summarize some core facts.

In 1960, the Philippines and South Korea had about the same standard of living, as measured by their per capita GDPs of about \$640 U.S. 1975. The two countries were similar in many other respects. ... From 1960 to 1988, GDP per capita in the Philippines grew at about 1.8 percent per year, about the average for per capita incomes in the world as a whole. In Korea, over the same period, per capita income grew at 6.2 percent per year, a rate consistent with the doubling of living standards every 11 years (Lucas, 1993, p. 251).

A number of researchers have theoretically or empirically addressed this sort of pattern, in which countries with similar economic fundamentals exhibit markedly different growth.¹ In particular, it is well established that growth disparity can be accounted for by using a dynamic general equilibrium model with growth (or convergence) path indeterminacy. This line of research generally assumes that diversified growth patterns arise during the transition process to the *uniquely* determined long-run equilibrium. However, a theory that admits the possibility of an economy reaching a *different* steady state in the long-run is more persuasive. To achieve such a model, dual steady states should be reachable in the global context. Therefore, a typical single-equilibrium model with multiple converging paths is somewhat inadequate for describing the episodes of disparate growth in South Korea and the Philippines discussed by Lucas (1993).

In these episodes, South Korea—a high-growth country—converged to the favored steady state by pursuing the *unexpected* and *extraordinary* path, whereas the Philippines—a low-growth country—converged to the relatively unsatisfactory steady state at a decent growth speed. These are qualitatively different. To state it plainly, a model to account for growth disparity of this type should be capable of replicating two contrasting situations simultaneously; a high-growth equilibrium that exhibits *indeterminacy* and a low-growth equilibrium that exhibits *saddle-path stability*. A graphical representation of this situation is shown in Fig. 1.

¹See, for instance, Benhabib and Perli (1994), Benhabib and Gali (1995), Cazzavillan (1996), Bennett and Farmer (2000), and others.

[Insert Fig. 1 around here]

The model we propose in this paper has a relatively simple structure that includes public health infrastructure as a factor.² Public health infrastructure is a factor that permits the existence of multiple steady states, and it therefore plays an essential role in our argument. In contrast to the bulk of the existing literature, we focus here on the case of multiple equilibria. Accordingly, we propose a model framework that can replicate the growth disparity noted above. A notable achievement of our framework is that indeterminacy of equilibrium paths around the high-growth steady state holds robustly under reasonable parameters.

The rest of the paper is organized as follows. Section 2 presents a basic model and shows multiplicity of equilibria in the global context under an assumption imposed on deep parameters. In Section 3, for the purpose of later numerical analysis, we clarify local dynamics properties of the model. Section 4 presents the results of numerical computations and shows that they are consistent with the selected episodes. In Section 5, we provide concluding remarks.

2. Model

Our model has some notable features with respect to model specifications. First, public health infrastructure boosts individual labor productivity in goods production. Second, the level of public infrastructure positively affects agent utility. Formally, a representative household maximizes (1) under the constraints of (2)–(4):

$$\max_C \int_0^{+\infty} \frac{(CH^\sigma)^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt, \quad \sigma \geq 0, \theta > 0, \theta \neq 1, \rho > 0, \quad (1)$$

subject to

$$\dot{K} = Y - C - G, \quad K(0) = K_0 > 0, \quad (2)$$

$$Y = K^\alpha (HL)^{1-\alpha}, \quad \alpha \in (0, 1), \quad (3)$$

$$G = \tau Y, \quad \tau \in (0, 1), \quad (4)$$

where C , H , K , Y , G , and L represent consumption, public health infrastructure, physical capital, output, government expenditure on public health, and labor, respectively.³ The deep parameters σ , $1/\theta$, and ρ are the weight of public health in the utility function, the elasticity of intertemporal substitution, and

²Though the directions of research are different and the model specifications are also slightly different, the qualitative properties for the case of single equilibrium have been elucidated by Capolupo (2000) and Hosoya (2003, 2005).

³We omit the time argument t .

the subjective discount rate, respectively. Parameters α and τ denote, respectively, the share of physical capital in goods production and the proportional income tax rate. The labor supply is assumed to be constant, and we have normalized to $L = 1$ throughout the paper; hence, all variables are per capita.

Equation (4) implies that government expenditure, G , is financed by income tax, τY , collected from private agents. The government balances its budget at each point in time. For a decentralized economy, H is an exogenous stock variable, and so each household maximizes its own utility by ignoring the effect of public health infrastructure.⁴ For a given level of public health infrastructure, an agent's dynamic optimization yields

$$g_C = \frac{1}{\theta} \left(\alpha(1 - \tau) \left(\frac{K}{H} \right)^{\alpha-1} + \sigma(1 - \theta)g_H - \rho \right), \quad (5)$$

where g_x denotes the growth rate of placeholder x .

Next, we examine the evolution of public health infrastructure. It is assumed that the infrastructure level is enhanced by government expenditure on public health and by a capital deepening externality. That is,

$$\dot{H} = \delta G \left(\frac{\bar{K}}{\bar{H}L} \right)^\epsilon, \quad \epsilon \in (0, 1),$$

where $\delta > 0$ is a constant efficiency parameter. This sort of specification is often employed in the growth literature for models with human capital and health infrastructure (e.g., Capolupo, 2000; Gupta and Barman, 2010). The right-hand side of this equation consists of two input factors: G and $\bar{K}/(\bar{H}L)$, where \bar{K} and \bar{H} are the society-average levels of physical and health capital, respectively. The ratio $\bar{K}/(\bar{H}L)$ represents the society-average level of private physical capital/effective labor ratio, which induces the external effects of capital deepening for public health creation. In other words, this factor corresponds to the societal average of the capital equipment ratio. The effect of this ratio is a social benefit derived from an improvement in living standards, and it is one of the most fundamental indicators of economic development.⁵

[Insert Fig. 2 around here]

To determine a statistical relation between health infrastructure (health status) level and living standards, we use infant mortality rate and per capita income as proxy variables. The relation shown in Fig. 2 is typical for various

⁴For this reason, a joint concavity condition imposed on C and H (i.e., $\theta \geq \sigma/(1 + \sigma)$) is not needed in the present case. Raurich (2003) and Agénor (2008) employ a qualitatively similar utility function.

⁵For a more detailed discussion of this argument, see Hosoya (2003).

countries.⁶ A close and negative correlation can be clearly confirmed. Moreover, our simple panel estimation provides additional evidence that supports this finding.⁷ On the basis of the results from the investigation above, our accumulation equation for public health infrastructure with a capital deepening externality is a reasonable specification.

Public health infrastructure is taken as *societal overhead capital*, so it should be specified as an exogenous variable for each agent. Consequently, the government bears the responsibility for health infrastructure provision through public expenditure. Since $L = 1$ was assumed, the following holds:

$$\dot{H} = \delta\tau K^\alpha H^{1-\alpha} \left(\frac{\bar{K}}{\bar{H}} \right)^\epsilon.$$

At equilibrium, \bar{K} and \bar{H} must be equal to K and H , respectively. Therefore, we obtain

$$g_H \equiv \frac{\dot{H}}{H} = \delta\tau \left(\frac{K}{H} \right)^{\alpha+\epsilon}. \quad (6)$$

Here, from (6), $K/H = (g_H/\delta\tau)^{1/(\alpha+\epsilon)}$. Since $g \equiv g_Y = g_C = g_K = g_H$ is satisfied at the balanced growth path (BGP), from (5), we obtain

$$g = \frac{1}{\theta} \left(\alpha(1-\tau) \left(\frac{g}{\delta\tau} \right)^{\frac{\alpha-1}{\alpha+\epsilon}} + \sigma(1-\theta)g - \rho \right). \quad (7)$$

From (7), we find that the equilibrium growth rate at the BGP depends on the set of parameters $\{\alpha, \tau, \delta, \epsilon, \sigma, \rho, \theta\}$.

Now, we make the following assumption for analytical purpose:

Assumption. $\theta < \sigma/(1+\sigma)$.

This leads by a straightforward process to the following proposition.

Proposition (The possibility of multiple equilibria). *Under the assumption $\theta < \sigma/(1+\sigma)$ on deep parameters, multiple equilibria are possible.* \square

Proof. From (7), $(\theta - \sigma(1-\theta))g + \rho = \alpha(1-\tau)(g/\delta\tau)^{(\alpha-1)/(\alpha+\epsilon)}$ is obtained. Now, we define the left-hand side of this equation by $\Psi(g)$ and the right-hand

⁶Fig. 2 covers 112 low- and middle-income countries in 2012. For the details of the two variables, see the next footnote.

⁷For 112 low- and middle-income countries over the period 1990–2012, a panel estimation with individual fixed effect yields

$$\begin{aligned} \ln MORTALITY &= 10.185 - 0.819 \ln GDP + \epsilon, \\ &\quad (0.436) \quad (0.055) \\ R^2 &= 0.631, \end{aligned}$$

where $MORTALITY$ is infant mortality rate (per 1,000 live births), GDP is GDP per capita, PPP (2005 international dollars using PPP rates), and ϵ is the error term. Standard errors are in parentheses.

side by $\Gamma(g)$. Then, $\Gamma(g)$ is a strictly decreasing and strictly convex function in the first quadrant of the (g, Γ) plane. Under the assumption that $\theta < \sigma/(1+\sigma)$, it is possible to represent Ψ by a linear function of g with a negative slope in the (g, Ψ) plane. Accordingly, given the shape of Γ , dual BGPs (equivalently, dual steady states) are possible except in the case where the two functions have no intersection. \blacksquare

We note the following interesting features before moving to the next section. The presence of public health infrastructure plays a critical role in the possibility of multiple steady states in this model. In fact, if $\sigma = 0$, then we can easily confirm that the equilibrium is uniquely determined and exhibits local saddle-path stability.⁸ We can also confirm that the presence of a capital deepening externality is not a necessary condition to obtain more than one equilibrium. Even for $\epsilon = 0$, dual steady states are likely to occur. Numerical analysis will be used to clarify how ϵ affects the growth rate of each equilibrium and local stability.

3. Local dynamics properties

By introducing new variables $X \equiv C/K$ and $Z \equiv K/H$, we can reduce the original three-dimensional system on C , K , and H to the two-dimensional system on X and Z . By using (2)–(4), (5), and (6), the following holds:

$$\frac{\dot{X}}{X} = X + \left(\frac{1-\theta}{\theta} \right) \delta \sigma \tau Z^{\alpha+\epsilon} + \left(\frac{\alpha-\theta}{\theta} \right) (1-\tau) Z^{\alpha-1} - \frac{\rho}{\theta}, \quad (8)$$

$$\frac{\dot{Z}}{Z} = -X + (1-\tau) Z^{\alpha-1} - \delta \tau Z^{\alpha+\epsilon}. \quad (9)$$

These equations characterize the dynamics of the model. Two conditions are needed to guarantee economically meaningful solutions. First, the per capita growth rate must satisfy the transversality condition, $\rho - (1-\theta)g > 0$. Second, the positivity condition on the consumption to physical capital ratio must be satisfied. As a result, from (9) with (6), g needs to satisfy $X^* = (1-\tau)(Z^*)^{\alpha-1} - g > 0$ in the steady state.⁹ In the following numerical analysis, these conditions are naturally satisfied.

To investigate local stability properties, the signs of the determinant ($\text{Det } J^*$) and the trace ($\text{Tr } J^*$) of the Jacobian must be inspected. From the corresponding Jacobian (see Appendix A), we have

⁸The saddle-path stability follows from (10), which is introduced later.

⁹The $*$ denotes the steady-state value. Also, if g is positive, then Z^* is automatically positive from (6).

$$\text{Det } J^* = -\frac{\alpha}{\theta}(1-\alpha)(1-\tau)X^*(Z^*)^{\alpha-1} - \left(\frac{\theta - \sigma + \sigma\theta}{\theta}\right)\delta\tau(\alpha + \epsilon)X^*(Z^*)^{\alpha+\epsilon}, \quad (10)$$

$$\text{Tr } J^* = X^* + (\alpha - 1)(1 - \tau)(Z^*)^{\alpha-1} - \delta\tau(\alpha + \epsilon)(Z^*)^{\alpha+\epsilon}. \quad (11)$$

The linearized system includes one control-like variable, X , and one state-like variable, Z . There are three possibilities for the local dynamics: (i) $\text{Det } J^* < 0$ (locally saddle-path stable); (ii) $\text{Det } J^* > 0$ and $\text{Tr } J^* > 0$ (locally unstable) and (iii) $\text{Det } J^* > 0$ and $\text{Tr } J^* < 0$ (locally indeterminate).

4. Numerical analysis

We will first explain benchmark parameters for numerical analysis. Although there is no consensus in the literature on the proper value for σ , we set a value to satisfy the equilibrium conditions based on Raurich (2003). In addition, θ must fulfilled some conditions, including the condition for multiple equilibria. For the value of θ that will do so, many earlier empirical studies have suggested a value larger than unity. As pointed out by Ben-Gad (2012), however, a lower value of θ has been confirmed as tenable in some recent studies, including Hansen et al. (2007). In this respect, the value that we use for θ is possibly justified. The values of both ρ and α are within a standard range of values for those parameters. Due to the specification of (4), the income tax rate, τ , can be seen as the ratio of public health expenditure to GDP. Therefore, we set this value according to empirical evidence.¹⁰ For δ and ϵ , as for σ , there is not yet consensus on the appropriate value. Accordingly, the values for these parameters are chosen so to induce the appropriate long-run growth rate in light of long-run time-series evidence. Table 1 represents our benchmark values.¹¹

[Insert Table 1 and Table 2 around here]

The two growth rates, the corresponding values of X^* and Z^* , the corresponding signs of $\text{Det } J^*$ and $\text{Tr } J^*$, and the corresponding results of equilibrium property are shown in Table 2. The steady-state growth rates for the high- and low-growth equilibria are about 3.4% and 1.7%, respectively. Probably, by searching among parameter constellations, we can approximate the growth rate at the high-growth equilibrium so that it corresponds to the observed average growth rate in South Korea. The sign of $\text{Det } J^*$ at the high-growth equilibrium

¹⁰See, for instance, *World Development Indicators*.

¹¹Incidentally, as noted before, the equilibrium is uniquely determined and exhibits local saddle-path stability when $\sigma = 0$. For example, when the same parameters in Table 1 are applied, except for σ , we obtain the growth rate of 1.16%.

is positive, and so either case (ii) or case (iii) (as described in the last part of the previous section) will be obtained. Therefore, we must inspect the sign of $\text{Tr } J^*$. Since $\text{Tr } J^* < 0$ is obtained from (11), the high-growth equilibrium is locally indeterminate. In contrast, the finding that $\text{Det } J^* < 0$ for low-growth equilibrium leads directly to low-growth equilibrium having local saddle-path stability.¹² This result is a key result of our investigation and worthy of special mention. First and foremost, when replicating the selected episodes of growth disparity, we can obtain a very accurate result.¹³

[Insert Table 3 around here]

To check the robustness of the result, let us change the value of the parameter ϵ , which is considered to have a marked effect on the dynamical system.¹⁴ The effect of this change as it is seen by the two growth rates of each steady state and the equilibrium properties are particularly noteworthy. Specifically, we update ϵ from 0.1 to 0.12 (i.e., a 20% increase) while keeping the other parameters at the same values shown in Table 1. The sensitivity result is displayed in Table 3 and is summarized as follows. First, the disparity between growth rates is substantially reduced as a result of the benefit from the capital deepening externality and the resulting accumulation of public health infrastructure. This is an important finding for public health and macroeconomic policies in developing countries. Because self-fulfilling expectations on the future provision of public infrastructure have a decisive influence on which equilibrium is attained, this finding has an important implication in dictating the orientation of the stabilization policy. More concretely, in addition to spending on public health, the government should promote provision of other types of societal capital (e.g., road maintenance, city park improvement, and river improvement) that results in a higher standard of living. Such efforts are effective for altering the inferior equilibrium under multiple equilibria. Second, on the local dynamics of the two steady states, those remain unchanged, as seen in Table 3. In consequence, we can declare that the simultaneous pursuit of indeterminacy and saddle-path stability, which deserves special mention in this paper, has a degree of robustness.

¹²This corresponds to case (i).

¹³As compared to the existing studies on related topics, including Benhabib and Perli (1994), Mino (2004), Park and Philippopoulos (2004), and Pérez and Ruiz (2007), our numerical analysis provides a novel result. In the models of Benhabib and Perli (1994, Section 4) and Park and Philippopoulos (2004), local equilibrium properties similar to our two equilibria are derived, but the growth rate at the inferior equilibrium (locally determinate) was practically zero in both studies. Consequently, these models are better suited to describe *poverty traps* than they are to describe our selected episodes.

¹⁴In light of the structure of the model, the change has an effect on the growth rates similar to that from a change in α .

5. Concluding remarks

Using a relatively simple model of economic growth, this paper has investigated theoretical methods of accounting for some well-known episodes of growth disparity, such as that between South Korea and the Philippines as discussed in Lucas (1993). Our numerical analysis indicates that the model presented above has a certain explanatory power for those episodes. More specifically, indeterminacy backcasts the remarkable growth miracle in South Korea, and saddle-path stability, which implies a less cyclic growth pattern, explains economic stagnation in the Philippines. In future research, we intend to work toward further improvement in consistency between theoretical and historical outcomes.

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Appendix A. Derivation of the Jacobian

From the reduced system of (8) and (9),

$$\begin{pmatrix} \dot{X} \\ \dot{Z} \end{pmatrix} = J^* \begin{pmatrix} X - X^* \\ Z - Z^* \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} X - X^* \\ Z - Z^* \end{pmatrix},$$

where

$$\begin{aligned} a_{11} &= \left. \frac{\partial \dot{X}}{\partial X} \right|_{\text{BGP}} = X^*, \\ a_{12} &= \left. \frac{\partial \dot{X}}{\partial Z} \right|_{\text{BGP}} = \left(\frac{1-\theta}{\theta} \right) \delta \sigma \tau (\alpha + \epsilon) X^* (Z^*)^{\alpha+\epsilon-1} \\ &\quad + \left(\frac{\alpha-\theta}{\theta} \right) (\alpha-1)(1-\tau) X^* (Z^*)^{\alpha-2}, \\ a_{21} &= \left. \frac{\partial \dot{Z}}{\partial X} \right|_{\text{BGP}} = -Z^*, \\ a_{22} &= \left. \frac{\partial \dot{Z}}{\partial Z} \right|_{\text{BGP}} = (\alpha-1)(1-\tau)(Z^*)^{\alpha-1} - \delta \tau (\alpha + \epsilon) (Z^*)^{\alpha+\epsilon}. \end{aligned}$$

The Jacobian, J^* , characterizes the local dynamics of the model.

References

- Agénor, P.-R., 2008. Health and infrastructure in a model of endogenous growth. *Journal of Macroeconomics* 30 (4), 1407–1422.

- Ben-Gad, M., 2012. The two sector endogenous growth model: an atlas. *Journal of Macroeconomics* 34 (3), 706–722.
- Benhabib, J., Gali, J., 1995. On growth and indeterminacy: some theory and evidence. *Carnegie–Rochester Conference Series on Public Policy* 43, 163–211.
- Benhabib, J., Perli, R., 1994. Uniqueness and indeterminacy: on the dynamics of endogenous growth. *Journal of Economic Theory* 63 (1), 113–142.
- Bennett, R.L., Farmer, R.E., 2000. Indeterminacy with non-separable utility. *Journal of Economic Theory* 93 (1), 118–143.
- Capolupo, R., 2000. Output taxation, human capital and growth. *Manchester School* 68 (2), 166–183.
- Cazzavillan, G., 1996. Public spending, endogenous growth and endogenous fluctuations. *Journal of Economic Theory* 71 (2), 394–415.
- Gupta, M.R., Barman, T.R., 2010. Health, infrastructure, environment and endogenous growth. *Journal of Macroeconomics* 32 (2), 657–673.
- Hansen, L.P., Heaton, J., Lee, J., Roussanova, N., 2007. Intertemporal substitution and risk aversion. In: Heckman, J.J., Leamer, E.E., (Eds.), *Handbook of Econometrics* 6A, Elsevier North-Holland, Amsterdam, pp. 3967–4056.
- Hosoya, K., 2003. Tax financed government health expenditure and growth with capital deepening externality. *Economics Bulletin* 5 (14), 1–10.
- Hosoya, K., 2005. The speed of convergence in a two-sector growth model with health capital. *PIE Discussion Paper Series No. 245*, Institute of Economic Research, Hitotsubashi University.
- Lucas, R.E., 1993. Making a miracle. *Econometrica* 61 (2), 251–272.
- Mino, K., 2004. Human capital formation and patterns of growth with multiple equilibria, in: Boldrin, M., Chen, B.-L., Wang, P. (Eds.), *Human Capital, Trade and Public Policy in Rapidly Growing Economies*. Edward Elgar, Cheltenham, pp. 42–64.
- Park, H., Philippopoulos, A., 2004. Indeterminacy and fiscal policies in a growing economy. *Journal of Economic Dynamics and Control* 28 (4), 645–660.
- Pérez, R., Ruiz, J., 2007. Global and local indeterminacy and optimal environmental public policies in an economy with public abatement activities. *Economic Modelling* 24 (3), 431–452.

Raurich, X., 2003. Government spending, local indeterminacy and tax structure. *Economica* 70, 639–653.

Table 1

Benchmark parameter values.

σ	θ	ρ	α	τ	δ	ϵ
3.5	0.45	0.065	0.35	0.03	0.13	0.1

Table 2

Equilibrium properties under multiple equilibria (benchmark).

Growth rate (%)	X^*	Z^*	Det J^*	Tr J^*	Result
3.40	0.009	122.973	+	−	indeterminate
1.73	0.099	26.354	−	...	saddle-point

Table 3

Equilibrium properties under multiple equilibria (sensitivity).

Growth rate (%)	X^*	Z^*	Det J^*	Tr J^*	Result
3.09	0.024	82.324	+	−	indeterminate
1.99	0.081	32.401	−	...	saddle-point

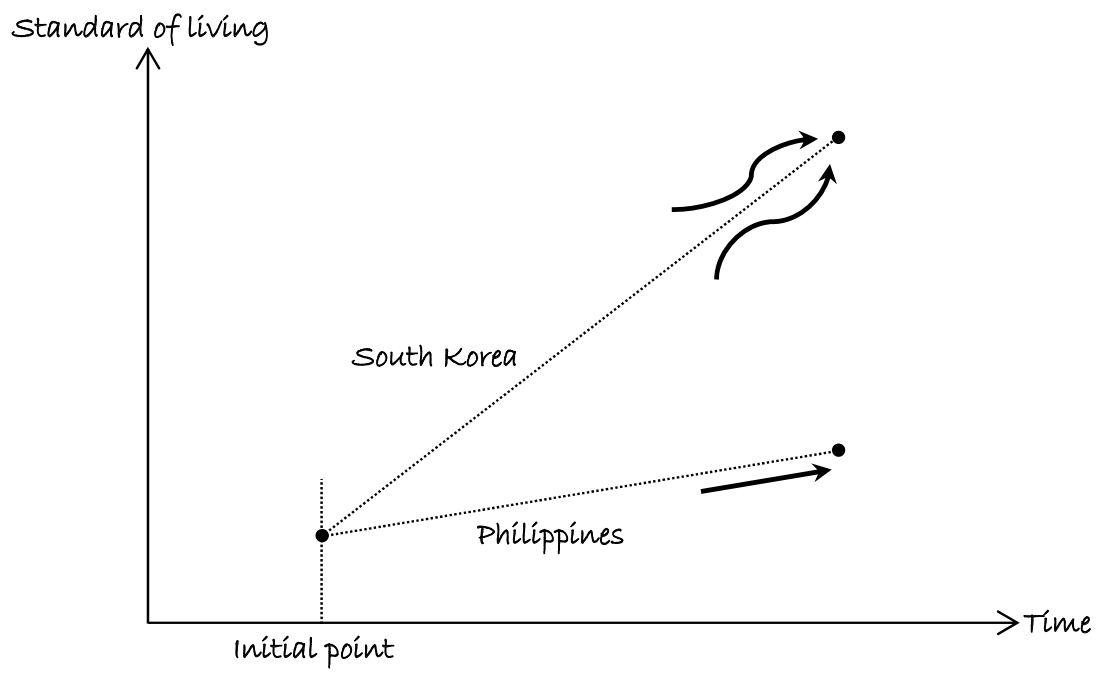
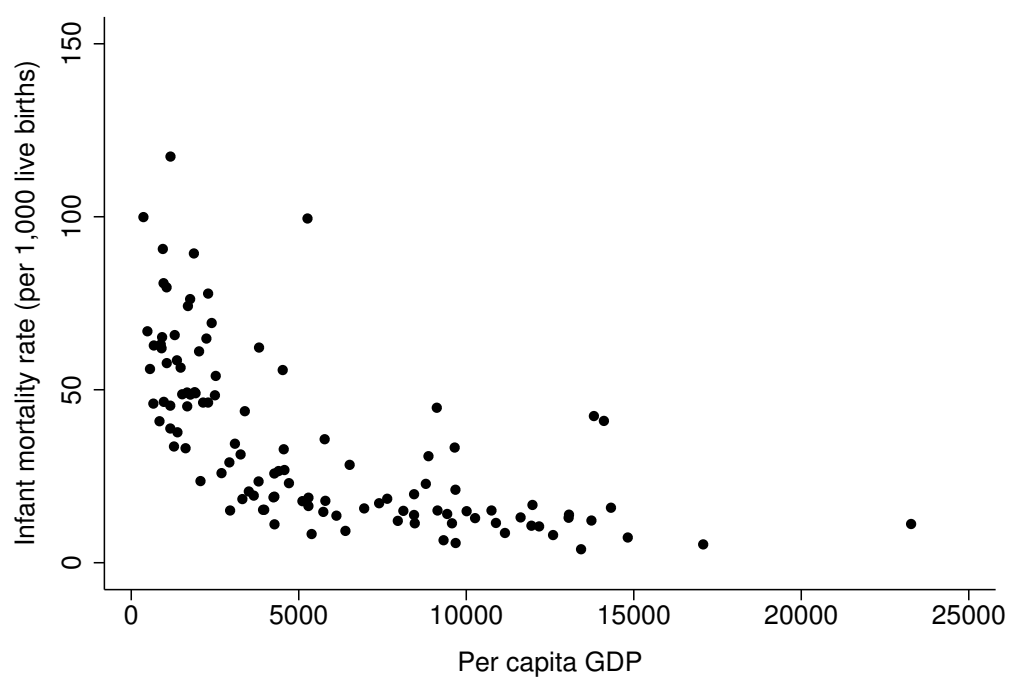


Figure 1: Growth disparity



Source: World Bank's *World Development Indicators*.

Figure 2: Per capita GDP, infant mortality rate