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Public Health Infrastructure and Growth: Ways to Improve the Inferior Equilibrium under Multiple Equilibria

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Abstract

This paper develops an endogenous growth model in which public health infrastructure, specified as a stock, plays an important role in economic growth. A notable feature of the model is that it employs a non-separable utility function for consumption, leisure and the level of public health. In addition, increasing the level of health infrastructure contributes to the production of goods through labor augmentation. With these settings, our model is found to have a unique equilibrium or multiple equilibria, depending on the magnitude of the intertemporal elasticity of substitution. For the case of multiple equilibria, we numerically study the ways to avoid the low-growth state in developing countries. From this, we identify two feasible policy implications. The results indicate that public health infrastructure has a vital role for the development policies in low-income countries. Lastly, we show that there are two possibilities in regards to the local dynamics of the model.

Keywords: Health infrastructure; Leisure; Multiple equilibria; Low-growth state

JEL classification numbers: I15; I18; O11; O41

1. Introduction

The provision of various public goods and services such as infrastructure has long been considered as having a vital role in promoting economic growth, in particular for developing countries. On the importance of public services and public capital, Arrow and Kurz (1970) and Barro (1990) made early and influential contributions, and subsequent theoretical and empirical studies have expanded in many different directions.[1]

The present paper focuses on the effects of public health capital and infrastructure on the process of growth and development. In the past decade, a number of researchers have given attention to the fact that good health is an indispensable prerequisite for growth, whereas growth contributes a great deal to good health.[2] Within a variety of social infrastructures, therefore, the importance of health infrastructure cannot be overstated. Several empirical studies confirm the fact that proxies for health are robust variables for explaining the subsequent growth in a country, and thus support the above view on the importance of health in economic development (Sala-i-Martin et al., 2004; Aghion et al., 2011).

Agénor (2008) and Gupta and Barman (2010) have recently contributed two important papers that directly examine the effects of health and its related factors on economic growth.[3] Agénor (2008) extends Barro (1990) and separately

[1] In line with the literature, we use *public capital* and *public infrastructure* interchangeably throughout the paper.

[2] In the modern context, we would say that the paper by van Zon and Muysken (2001) is a pioneering study in this field of research. At the same time, theoretical studies have made progress in step with developments in endogenous growth theory and intriguing empirical studies have begun to appear. López-Casasnovas et al. (2005) and Garibaldi et al. (2010) provide a glimpse of the developments in this area.

[3] Utilizing the framework of overlapping-generations (OLG), Bhattacharya and Qiao (2007) and Varvarigos and Zakaria (2013) construct a growth model with public and private expenditures on health. In these models, public expenditures complement private spending. Recently, Agénor (2013) presented comprehensive research on public policy using the OLG models.

incorporates health services (flow or stock) and infrastructure services into the model. Therefore, this model investigates the relationship between health, infrastructure and growth, and derives implications for the allocation problem of government investment between health and infrastructure.[4] For our purposes, the second model of Agénor (2008), which treats health as a stock variable, is especially important to consider. In this model, a unique steady-state equilibrium satisfies saddle-path stability.

Gupta and Barman (2010) extend the model of Agénor (2008). By introducing environmental quality to the Agénor's framework, they investigate optimal fiscal policy in the steady-state equilibrium, and then clarify the dynamic properties of the equilibrium.[5] Accordingly, the model exhibits local indeterminacy if certain conditions on the dynamic system are satisfied, since the environment and health stocks generate production externalities.

Raurich (2003) is also important to consider in the model specification. Although public input is treated as a flow variable, it is embedded in both the utility and production functions, and allows for an elastic labor supply.

On the basis of this avenue of research, we develop an endogenous growth model with publicly provided health infrastructure, and examine its policy implications and the properties of its equilibrium dynamics.[6] The characteristics

[4]This paper has three key characteristics: (i) the production functions for final goods and health services include public infrastructure spending; (ii) the level of health services is an input in goods production, while at the same time health spending is an input in health production; and (iii) the agent's utility is affected by health service level.

[5]In comparison with Agénor (2008), beyond the environmental factor, the following three points are worth noting: (i) the production function for final goods is essentially the same as Agénor (2008), though it considers the absolute congestion effect on infrastructural input; (ii) health capital (infrastructure) is accumulated through expenditure on health only; and (iii) the utility function is composed of consumption only, for analytical simplicity.

[6]One of the notable features of the model is that it employs a non-separable utility function between consumption, leisure and the level of public health infrastructure. In addition to those above mentioned, our specification is similar to the one employed by Bennett and Farmer (2000) and Fernández et al. (2004), among others.

of our model structure relative to Agénor (2008), Gupta and Barman (2010), Raurich (2003) and others are as follows. First, we specify public health infrastructure as a stock variable, unlike with Raurich (2003); thus, the production function for final goods includes the health stock in a labor augmenting manner.[7] Second, in the model, health infrastructure evolves through government health investment only, as in Gupta and Barman (2010). Third, the agent's utility function includes the leisure time fraction and exogenously given health infrastructure levels, in line with Raurich (2003). The labor–leisure choice is not considered in Agénor (2008) and Gupta and Barman (2010), but the introduction of the choice is crucial for a dynamic general equilibrium analysis. Fourth, we allow a larger value for the intertemporal elasticity of substitution, which differs from previous studies in the literature, and is an important aspect of our equilibrium properties.

We obtain several interesting results from the solution of the model. The model has a unique steady-state equilibrium when there is a smaller value for the intertemporal elasticity of substitution, while a larger value yields multiple steady states. Therefore, the agent's preference influences the number of steady-state equilibria. In previous growth models with health infrastructure, little attention has been given to the case of multiple equilibria; hence it seems that our exploration makes a valuable contribution to the literature.

To achieve concrete policy implications, we experiment with numerical analysis for the analytical solution. Namely, we observe the effects on growth rates by changing parameters (e.g., the preference, technology and policy parameters).[8] Compared with the case of a single steady state, there is a different picture in the case of dual steady states. Moreover, in the experiment we take a particular interest in identifying ways for developing countries to improve the

[7]As pointed out in Agénor (2008), making a distinction between public infrastructure and health capital is essential; however, we do not differentiate these inputs for simplicity.

[8]Preference parameters in the utility function, such as the rate of time preference, are called *deep* parameters.

inferior equilibrium in terms of growth rate, which are examined in detail. We present specific policy solutions, with consideration given to their feasibility.

Lastly, the local dynamics properties of the model are investigated. Because of the complexity of our dynamic system, we are forced to attempt an analysis while employing some important assumptions. As a result, we show that there are two possibilities for the local dynamics. The first is the case where the equilibrium is locally saddle-path stable.[9] The second is the case where the equilibrium is locally indeterminate. In the present context, local indeterminacy implies that there is a continuum of equilibrium paths converging to a given steady state.[10] Supplemental numerical computation implies that saddle-path stability is likely to be found.

The rest of the paper is organized as follows. Section 2 presents the basic framework of the model and its long-run equilibrium solution, and examines the equilibrium properties. Section 3 presents numerical computations for the theoretical results. In Section 4, we summarize the dynamic system and analyze the local stability of the dynamic equilibrium. Section 5 provides concluding remarks.

2. The model

2.1. Basic framework

In this section, we present a simple growth model that includes public health infrastructure. One infinitely lived representative household derives utility from consumption, C, leisure time, l, and the level of public health infrastructure, H, in the macroeconomy.[11] The household's intertemporal utility is specified

[9] As noted before, the same result is obtained in Agénor (2008).

[10] A number of authors have examined the properties of equilibrium (path) indeterminacy in neoclassical and endogenous economic growth models. Typical examples include Benhabib and Perli (1994) and Mino (2001).

[11] For analytical simplicity, the overall population is always constant and normalized to unity ($L = 1$) such that all variables become per capita amounts. The public health aspects in this paper are discussed further in the next subsection.

as

$$\int_0^{+\infty} e^{-\rho t} U(C(t), l(t), H(t)) \mathrm{d}t,$$

where $\rho > 0$ is the rate of time preference.[12] The instantaneous utility is assumed to be given by

$$U(C, l, H) = \frac{(C l^{\eta} H^{\sigma})^{1-\theta} - 1}{1-\theta}, \quad \eta > 0, \ \sigma > 0, \ \theta > 0, \tag{1}$$

where $1/\theta$ is the elasticity of intertemporal substitution, and η and σ are the weights of leisure and public health infrastructure in the household's utility function, respectively. Note that in a decentralized economy, H is an exogenous variable; thus, the household maximizes its own utility ignoring the effect of public health infrastructure. Therefore, the joint concavity condition imposed on C and l is $\theta \geq \eta/(1+\eta)$.

The household is endowed with one unit of time that can be devoted to either work or leisure (i.e., $1 - l$ is the unit of time devoted to work). Because the government imposes a tax on total household income at a rate τ, the household's budget constraint without physical capital depreciation becomes

$$\dot{K} = (1-\tau)(rK + w(1-l)H) - C, \tag{2}$$

where K is physical capital, and r and w are the interest rate and the wage rate, respectively.[13] Here, it can be found that public health infrastructure, H, boosts labor productivity, thus health is *labor augmenting*. The production function of the representative firm is specified as the standard Cobb–Douglas technology:

$$Y = K^{\alpha}((1-l)H)^{1-\alpha}, \quad \alpha \in (0, 1). \tag{3}$$

Profit maximization yields

$$r = \alpha \frac{Y}{K} \tag{4}$$

and

$$w = (1-\alpha)\frac{Y}{(1-l)H}. \tag{5}$$

[12]From now on, we will suppress the time argument when not needed for clarity.

[13]As noted before, total labor force is normalized to unity.

2.2. The evolution of public health infrastructure

The factor of *health* is one of the key elements in this study, and is specified as public health infrastructure. Good health is indispensable to human life and is of particular importance in developing countries. For each agent, public health infrastructure is considered as social overhead capital, so it is appropriate that the household takes H as an exogenous variable. Consequently, the government bears the responsibility for health infrastructure provision.

In line with our specification, income tax revenue from the household is devoted to the improvement of public health, thus H is treated as a stock variable. Assuming that the government maintains a balanced budget at each point in time ($G = \tau Y$) and all the tax revenue is associated with public health expenditures, the accumulation of health infrastructure is determined by

$$\dot{H} = \delta G = \delta \tau Y = \delta \tau K^{\alpha}((1-l)H)^{1-\alpha}, \quad \delta > 0, \tag{6}$$

where δ is a technological efficiency parameter. A similar specification is often employed in the related literature, including Capolupo (2000), Gupta and Barman (2010) and Hosoya (2012).

To sum up the specifications, there are two influential factors for agent's utility apart from consumption. The first is leisure activities; the second is the provision of public health infrastructure. As leisure time decreases and labor activities increase, income increases and the tax base is expanded. In the macroeconomy, abundant tax revenue leads to the improvement of the infrastructure level. Although the provision of public health infrastructure is an external factor for individual agents, this externality indirectly increases their own utility levels.

2.3. Solution for the decentralized economy

The optimal solution for the decentralized economy is to choose at each moment in time the amount of consumption, $\{C\}_{t=0}^{+\infty}$, the amount of physical capital, $\{K\}_{t=0}^{+\infty}$, and time fractions allocated to production and leisure activities.

Given budget constraint (2), the initial stock of physical capital $K(0) = K_0$, the factor prices r and w and fiscal policy variable τ, the household maximizes the infinite stream of discounted instantaneous utility (1).

To solve the household's optimization problem, we formulate the current-value Hamiltonian, $\mathcal{H}$:

$$\mathcal{H} \equiv \frac{(Cl^{\eta}H^{\sigma})^{1-\theta} - 1}{1-\theta} + \lambda[(1-\tau)(rK + w(1-l)H) - C],$$

where λ is the co-state variable associated with constraint (2). The first-order necessary conditions are given by

$$\frac{(Cl^{\eta}H^{\sigma})^{1-\theta}}{C} = \lambda, \tag{7}$$

$$\frac{\eta(Cl^{\eta}H^{\sigma})^{1-\theta}}{l} = \lambda(1-\tau)wH, \tag{8}$$

$$(1-\tau)r - \rho = -\frac{\dot{\lambda}}{\lambda}, \tag{9}$$

plus the usual transversality condition

$$\lim_{t\to+\infty} \lambda(t)K(t)e^{-\rho t} = 0. \tag{10}$$

When the transversality condition (10) holds, the necessary conditions (7)–(9) are also sufficient under resource constraint (2).

By log-differentiating (7) and applying (9), we obtain the Keynes–Ramsey rule, taking account of the labor–leisure choice and the existence of public health infrastructure:

$$\frac{\dot{C}}{C} = \frac{1}{\theta}\left((1-\tau)r - \rho + \eta(1-\theta)\frac{\dot{l}}{l} + \sigma(1-\theta)\frac{\dot{H}}{H}\right). \tag{11}$$

From (7) and (8), we arrive at

$$\frac{l}{C} = \frac{\eta}{(1-\tau)wH}. \tag{12}$$

An equilibrium solution for the decentralized economy is essentially characterized by (11) and (12).[14] From (11), we find that it is necessary to clarify

[14]Note that r and w are represented by (4) and (5).

the dynamics of leisure time and public infrastructure. As for $\dot{l}/l$, the following equation can be derived by using (6) (see Appendix A):

$$\frac{\dot{l}}{l} = \frac{\dot{C}}{C} - \alpha(1-\tau)(1-l)^{1-\alpha}\left(\frac{K}{H}\right)^{\alpha-1} + \alpha\frac{C}{K} - \alpha\frac{\dot{l}}{1-l} - (1-\alpha)\frac{\dot{H}}{H}. \tag{13}$$

Here, as for $\dot{H}/H$, we simply use the transformed expression of (6). That is,

$$\frac{\dot{H}}{H} = \delta\tau(1-l)^{1-\alpha}\left(\frac{K}{H}\right)^{\alpha}. \tag{14}$$

In view of (14) and the expression of $r = \alpha(1-l)^{1-\alpha}(K/H)^{\alpha-1}$, substituting (13) into (11) yields:

$$\begin{aligned}(\theta - \eta(1-\theta))\frac{\dot{C}}{C} \\ &= (1-l)^{1-\alpha}Z^{\alpha}\left(\frac{\alpha(1-\tau)(1-\eta(1-\theta))}{Z} + \delta\tau(1-\theta)(\sigma - \eta(1-\alpha))\right) \\ &+ \alpha\eta(1-\theta)\left(X - \frac{\dot{l}}{1-l}\right) - \rho,\end{aligned} \tag{15}$$

where $X \equiv C/K$ and $Z \equiv K/H$.

2.4. Long-run steady-state equilibrium

We now proceed to characterize the long-run (steady-state) equilibrium of the model. In our model, the long-run equilibrium is defined as follows:

Definitions 1. In the long-run steady-state equilibrium, C, K, H and Y grow at the same constant growth rate and the time fraction variable l remains at a constant value (i.e., $\dot{l} = 0$). Therefore, X and Z are also constant. Accordingly, our notations of each steady-state value are as follows: $\dot{C}/C = \dot{K}/K = \dot{H}/H = \dot{Y}/Y = g$, $l = l^*$, $X = X^*$ and $Z = Z^*$.

Applying the condition $\dot{l} = 0$, together with the above notations, (15) becomes

$$\begin{aligned}(\theta - \eta(1-\theta))g \\ &= (1-l^*)^{1-\alpha}(Z^*)^{\alpha}\left(\frac{\alpha(1-\tau)(1-\eta(1-\theta))}{Z^*} + \delta\tau(1-\theta)(\sigma - \eta(1-\alpha))\right) \\ &+ \alpha\eta(1-\theta)X^* - \rho.\end{aligned} \tag{16}$$

At the steady state, X^* and Z^* can be represented by the deep and technological parameters, l^* and g. Using (13) or the accumulation equation for physical capital, the consumption/physical capital ratio, X^*, is described by the following equation:

$$X^*(\alpha, \tau, g, l^*, Z^*) = (1-\tau)(1-l^*)^{1-\alpha}(Z^*)^{\alpha-1} - g. \tag{17}$$

On the other hand, the accumulation equation for public health infrastructure, (14), leads to the physical capital/public infrastructure ratio, Z^*. That is,

$$Z^*(\alpha, \delta, \tau, g, l^*) = \left(\frac{g}{\delta\tau(1-l^*)^{1-\alpha}}\right)^{\frac{1}{\alpha}}. \tag{18}$$

To obtain a positive X^*, the right-hand side of (17) must be positive.[15] All numerical analyses developed in this paper satisfy this condition.

Substituting (17) and (18) into (16), and then rearranging it, we get

$$\alpha(1-\tau)\left(\frac{g}{\delta\tau(1-l^*)}\right)^{\frac{\alpha-1}{\alpha}} = (\theta - \sigma(1-\theta))g + \rho, \tag{19}$$

where $g = g^*$, which satisfies (19), becomes the growth rate at the long-run equilibrium for positive values of X^*.[16] [17] In the following, we begin by investigating long-run equilibrium properties.

2.5. Equilibrium properties

To clarify the long-run equilibrium properties, we explore (19) in detail. Let the left-hand side (LHS) and the right-hand side (RHS) of (19) be denoted as

[15]A detailed investigation of this point was conducted by Gaspar et al. (2014).

[16]The growth rate of the economy can be written as a function of six parameters and l^*: $g = f(\theta, \sigma, \rho, \alpha, \delta, \tau, l^*)$.

[17]If the agent has a logarithmic utility function (i.e., $\theta = 1$), (19) is changed by

$$\alpha(1-\tau)\left(\frac{g}{\delta\tau(1-l^*)}\right)^{\frac{\alpha-1}{\alpha}} = g + \rho.$$

Γ and Ψ, respectively:

$$\Gamma(g) \equiv \alpha(1-\tau)\left(\frac{g}{\delta\tau(1-l^*)}\right)^{\frac{\alpha-1}{\alpha}}, \tag{20}$$

$$\Psi(g) \equiv (\theta - \sigma(1-\theta))g + \rho, \tag{21}$$

where both can be seen as a function of g. Now, we should check the functional properties. On the one hand, from (20), we find that $\Gamma(g)$ is a strictly decreasing and strictly convex function of g, in view of the functional properties $\lim_{g\to 0}\Gamma(g) = +\infty$, $\lim_{g\to 0}\Gamma'(g) = -\infty$, $\lim_{g\to+\infty}\Gamma(g) = 0$ and $\lim_{g\to+\infty}\Gamma'(g) = 0$. On the other hand, from (21), $\Psi(g)$ is a simple linear function of g, but the slope varies depending on the magnitude of θ. Specifically, there are three possibilities: first, when $\theta > \sigma/1+\sigma$, Ψ has a positive slope with intercept $\rho > 0$ in the relevant quadrant; second, when $\theta = \sigma/1+\sigma$, Ψ comes to a horizontal line at ρ; and third, when $\theta < \sigma/1+\sigma$, Ψ has a negative slope with intercept ρ.

According to the shapes of Γ and Ψ, we can state the following two propositions.

Proposition 1. *When $\theta \geq \sigma/(1+\sigma)$ and $\theta \geq \eta/(1+\eta)$, there exists a unique long-run equilibrium solution.*

The latter condition guarantees joint concavity concerning consumption and leisure time in the household's utility function. For example, the case where $\theta = 1$ corresponds to this scenario. Since the Ψ function has a positive or zero slope in the (g, Ψ)-plane, the equilibrium growth rate is uniquely determined.

Proposition 2. *When $\eta/(1+\eta) \leq \theta < \sigma/(1+\sigma)$, the emergence of multiple (dual) long-run equilibria is possible.*

To understand the statement of Proposition 2, we explore it further. Note that, in the present case, the Ψ function has a negative slope in the (g, Ψ)-plane. Consequently, (i) when the LHS (Γ) is located below the RHS (Ψ) over the whole range of positive g, no solution exists; (ii) when the LHS is tangent to the RHS, that is, $\Gamma'(g^*) = \Psi'(g^*)$, a single solution exists; and (iii) when the LHS is located above the RHS, two solutions exist. Case (iii) implies the emergence of dual steady states where two long-run growth rates exist that satisfy the optimality criteria. As might be expected, our current interest is the last case.

Now, the reason for the emergence of multiple equilibria can be explained as follows. Private agents (i.e., consumers and firms) in a decentralized economy cannot know the evolution of public health infrastructure. Therefore, if agents expect a high level of (productive) public health infrastructure, then they will increase their physical capital. This leads to higher income and higher tax revenue. Through these processes, a higher level of public health infrastructure is actually supplied, which results in the high-growth equilibrium. On the other hand, in the case where negative expectations are formed about the future level of infrastructure, the economy arrives at the low-growth equilibrium because of the poor level of public health provision.

On the basis of these observations, we will present several numerical computations in a later section.

3. Numerical computations

In this section, we further study numerical simulations for the theoretical results obtained in the previous analysis. Two theoretical possibilities are presented that depend on the deep parameter combinations. Let us begin with the case in which the long-run steady-state growth rate is uniquely determined.

3.1. The case of a single steady state

As shown in Proposition 1, when a relatively small value for the intertemporal elasticity of substitution holds (i.e., $\theta > \sigma/(1+\sigma)$), the rate of economic growth at the steady state is uniquely determined.

To satisfy the above condition for the preference parameters, we first set $\theta = 1.2$ and $\sigma = 0.8$. Following Stokey and Rebelo (1995), Gómez (2008) and others, the parameterization for θ can be considered reasonable. As for σ, we can assume that it takes either a value less than one or greater than one. For example, the seminal paper by Raurich (2003) employs 0.65–1.5 as the weight for public goods in the utility function.[18] Consequently, we test several cases. For the rate of time preference, ρ, the elasticity of physical capital in the production of goods, α, and the fraction of time devoted to leisure activities at the steady state, l^*, well-known standard values in the literature are employed (see, for example, Ladrón-de-Guevara et al., 1997; Ortigueira and Santos, 1997; Greiner, 2008). The income tax rate, τ, essentially follows empirical evidence.[19] The technological efficiency for the accumulation process of public health infrastructure, δ, is chosen so as to acquire the appropriate long-run growth rate in light of long-run time series evidence.[20] Our benchmark parameters are shown in Table 1.

[Insert Table 1 around here]

[Insert Fig. 1 (caption: Benchmark case (single steady state)) around here]

Fig. 1 shows the benchmark case. In this case, the long-run growth rate is 1.4%; hence, it proves the validity of our assumed scenario in light of long-run time series on per capita growth rate. To confirm the effects on growth rates from a change in each parameter, we observe the sensitivity of the benchmark

[18] As explained in Section 1, the frameworks of the present model and Raurich (2003) are similar in that they both include leisure and publicly provided goods.

[19] Due to the specification of (6), the tax rate τ can be seen as the ratio of public health expenditure to GDP. According to the *World Development Indicators 2012*, averaged public health expenditure (% of GDP) in 217 countries during 2000–2010 was 3.82%. Based on this evidence, we set a suitable value.

[20] Incidentally, as a technology parameter for human capital formation through schooling activities, the pioneering research by Lucas (1988) employed 0.05. Also, the recent paper by Greiner (2008) set the parameter at 0.15.

case.[21] Roughly speaking, compared with the benchmark case, the changes in τ and δ have a relatively significant influence on the growth rates. That is to say that an increase in both parameters boosts economic growth.[22] By these observations, in the case in which the equilibrium is uniquely determined at least, the government bears the responsibility of raising sufficient tax revenue and implementing efficient provision processes to develop a good public health environment. As will be clarified later, the same implication is true in the case of multiple equilibria.

3.2. The case of dual steady states

In Proposition 2, we demonstrated that a relatively large value for the intertemporal elasticity of substitution (i.e., $\eta/(1+\eta) \leq \theta < \sigma/(1+\sigma)$) yields the possibility of multiple equilibria (dual steady states). As pointed out by Ben-Gad (2012), a higher value of $1/\theta$ has often been obtained in some recent empirical studies including Hansen et al. (2007). In line with the single equilibrium case, we first to attempt a benchmark simulation under the present scenario. The benchmark parameters are listed in Table 2. To obtain economically meaningful and plausible dual steady states, we change some parameter values from the previous single equilibrium case. The rate of time preference, 0.1, is higher than typical of values used in the literature; nevertheless, it is a reasonable value. Park and Philippopoulos (2004), for instance, employ the same value and Greiner and Hanusch (1998) adopt a value higher than 0.1. The fraction of time devoted to leisure activities, 0.75, is also higher than before. However, calibration analyses, including Turnovsky (2002), Gómez (2008) and Pintea (2010), suggest that this is an acceptable value.[23] The parameters δ and

[21] The detailed numerical results from single equilibrium cases are available from the author upon request.

[22] The result for δ is basically the same as the famous two sector model of Lucas (1988), on the grounds that an increase in the production efficiency of the *second* sector contributes to long-run growth.

[23] Values of around 0.7 were either obtained or assumed in these studies.

σ are adopted so as to replicate the long-term annual growth rate. In comparison with the single equilibrium case, in particular, we find that a sizable value of σ is required in order to generate multiple equilibria.

[Insert Table 2 around here]

In setting parameter values, we need to satisfy the two conditions. Namely, (i) the above noted inequality condition that integrates the conditions for generating multiple equilibria and the joint concavity associated with the utility function, and (ii) the transversality condition.[24] So that the transversality condition (10) holds, we find it necessary to satisfy the following inequality:

$$\rho - (1-\theta)g > 0. \tag{22}$$

The case of a single steady state always satisfies (22) since $\theta > 1$.

Our procedure here is as follows. We first set θ and σ in light of condition (i), and then proceed to specific computations. In turn, for the growth rates of the two equilibria, we check (22) under each scenario. Naturally enough, all the examples we present below meet these requirements.

[Insert Fig. 2 (caption: Benchmark case (dual steady states)) around here]

We now move on to concrete analyses. Fig. 2 illustrates the reference case applying the benchmark parameters of Table 2. At the steady state, the two growth rates are 2.11% and 4.05%, respectively. The growth gap between the two equilibria is about 2%, so there is a considerable difference depending on which equilibrium is realized. As noted before, for our model economy, the

[24] For condition (i), η is free to be set such that the solutions of the model are not bound by the size of $\eta/(1+\eta)$. Therefore, we should focus attention on the relationship between θ and σ.

most descriptive country group is developing countries. Even under the low-growth steady-state equilibrium, the calculated growth rate (2.11%) is close to the average annual growth rate of U.S. per capita GDP over the last 140 years, which for these developing countries would constitute acceptable performance.[25] In a general understanding, when self-fulfilling expectations on the provision of public health infrastructure enter into a virtuous cycle, that will lead to a more rapid growth (high-growth steady state). To be assured of the positivity of X^* under a given set of parameters, we need the real economic growth rate to be less than or equal to 4.3%. Of course, the two growth rates satisfy this condition.[26]

In the following, we observe the effects of variations in τ and δ on growth rates because these parameters played a key role in the unique equilibrium case. Moreover, the impact of changes in leisure time fraction, l^*, and the elasticity of the utility of public health infrastructure, σ, are concretely examined.

[**Insert Fig. 3 (caption: A change in τ from 0.05 to 0.055 (dual steady states)) around here**]

Let us now change the income tax rate and efficiency parameter for public health. It was observed in the single equilibrium case that changes in these parameters have a significant influence on growth rate. Given this, Fig. 3 first shows the case of changing τ from 5% to 5.5%. Compared with the benchmark case, the growth rate at the high-growth steady state somewhat decreases to 3.74%, whereas the growth rate at the low-growth steady state increases to 2.52%.[27] It is worthwhile to note that the famous *nonmonotonic* relationship between public spending and economic growth derived by Barro (1990) can be observed in the two steady states at the same time. In particular the fact

[25] See Jones (2011, pp. 49–50)

[26] As noted before, all numerical explorations also satisfy the positivity condition.

[27] The latter case is notable, as we have confirmed a positive growth effect.

that a tax policy can remedy the inferior equilibrium is valuable knowledge for macroeconomic development policies. This point will be examined further in the following subsection. Secondly, we update δ from 0.65 to 0.7. The effects of this change on the equilibrium growth rates are similar to the case of the change in tax rate and this case is shown in Fig. 4. Compared with the benchmark, as expected, the increase in the growth rate at the low-growth steady state is of particular note ($g^*_{high} = 0.0382$, $g^*_{low} = 0.0242$).

[Insert Fig. 4 (caption: A change in δ from 0.65 to 0.7 (dual steady states)) around here]

From a theoretical point of view, a highlight of this paper is the introduction of elastic labor supply (i.e., labor–leisure choice). In the single equilibrium case, a change in the time fraction of labor supply (or leisure) had a certain impact on growth rate. In the case of multiple equilibria, the change also affects the two steady states, which is highly intriguing. Namely, self-fulfilling expectations with respect to the provision of public infrastructure, which lead to either the high- or low-growth equilibrium, are attenuated by an increase in labor supply. Fig. 5 corresponds to this case.

[Insert Fig. 5 (caption: A change in l^* from 0.75 to 0.73 (dual steady states)) around here]

When changing l^* from 0.75 to 0.73, we obtain growth rates of 3.81% and 2.43% for the high- and low-growth steady states, respectively. In comparison with the benchmark, the growth rate at the high-growth steady state decreases, whereas the growth rate at the low-growth steady state increases. As described before, if the agent expects a lower (resp. higher) provision of infrastructure, physical capital accumulation is dampened (resp. promoted). An increase in labor supply moderates the present situation since the gap in the two growth rates diminishes. This result implies that, even in developing countries, it is important to deal with various problems in the labor market, including the problem of unemployment, given that a change in labor supply influences growth rate.

[Insert Fig. 6 (caption: A change in σ from 3.7 to 3.85 and 4.0 (dual steady states)) around here]

Lastly for the case of multiple equilibria, we investigate the effects of a change in σ on a couple of growth rates. We develop the analysis in Fig. 6 in which different cases (including the benchmark case of Fig. 2) are displayed all together. A larger value of σ, which corresponds to the case where the agent benefits to a greater extent from public health infrastructure levels (exogenously given), appears to harm the growth rate at the high-growth steady state in the course of formulating one's own utility. Comparing with respect to g^*_{high} among the three cases, we can observe notable differences: 4.05% (benchmark; $\sigma = 3.7$), 3.74% ($\sigma = 3.85$) and 3.42% ($\sigma = 4.0$). The results can be explained as follows. An economy arriving at the high-growth steady state has accumulated more physical capital, thus investment in capital is less productive in this state. As σ increases, the importance of K to H is further diminished.[28] In such an economy with a higher level of development, as σ increases, long-run growth rate declines because individuals are assumed to prefer consumption to saving, whereas in the case of the low-growth equilibrium we cannot find significant variations in the growth rates.[29] Roughly speaking, when economically meaningful dual steady states arise, it can be said that a change in σ has a relatively small effect on the low-growth equilibrium.

3.3. Improving the inferior equilibrium under multiple equilibria

[28] At the high-growth equilibrium, as σ increases, X^* increases and Z^* decreases.

[29] The growth rates at the low-growth equilibrium are 2.11% (benchmark; $\sigma = 3.7$), 2.18% ($\sigma = 3.85$) and 2.29% ($\sigma = 4.0$), respectively.

How can we deliver a better growth process under emerging multiple equilibria? Is there the role for economic policy in stepping up to this challenge? If so, what is it exactly? In the last part of the numerical analysis, we focus on these problems.

In our model, the direction and degree of self-fulfilling expectations for future public infrastructure level show which equilibrium the economy will reach at the steady state. Consequently, a variety of policies are thought to have an indirect effect, whereas policies that have *direct* effects on equilibrium determination are likely limited. Now we define *desirable economic circumstances* under multiple equilibria from the viewpoint of growth rate as follows.

Definition 2. Under multiple equilibria, desirable economic circumstances are defined as the situation in which the growth gap between the high- and low-growth steady state is narrow.

Since an important role of macroeconomic policy is to stabilize economic fluctuations in general, this is a reasonable definition. The focus of our analysis here is *developing* countries and we apply the definition to such economies. Within the overall numerical results attempted in this paper, the case we will investigate below should meet both the requirements of feasibility and policy validity.[30] In this respect, we focus on two areas of economic policy, namely, changes in the income tax rate and the production efficiency of public health infrastructure.

[Insert Fig. 7 (caption: Improving the low-growth state (tax policy impact)) around here]

Fig. 7 shows the case where τ is increased from 5% to 6%. Note that σ is

[30] It is likely a difficult challenge for economic policies to directly affect agent's deep parameters.

also changed slightly from the benchmark case (σ=3.6).[31] Since the two growth rates are 3.4% (the high-growth steady state) and 3.13% (the low-growth steady state), regardless of which equilibrium is attained, no considerable difference is seen in comparison with the benchmark. A higher tax rate might well lead to a reduction in the gap, but for developing countries setting tax levels too high is unrealistic. In view of this, 6% is appropriate. Now, let us apply the well-known Rule of 70 to the present case. Thus, the number of years it takes for income to double is about 20.6 years at the high-growth steady state and 22.4 years at the low-growth steady state, respectively. We may conclude that there is no significant difference between the cases, because the difference is only about 2 years, a very slight difference.

[**Insert Fig. 8 (caption: Improving the low-growth state (public health policy impact)) around here**]

Fig. 8 represents the case in which δ is increased from 0.65 to 0.77. As for σ, we use the same values as above. In this scenario, the growth rate at the high-growth steady state is 3.53%, whereas the growth rate at the low-growth steady state is to 2.99%. As in the previous case, applying the Rule of 70, we can confirm that the difference in the number of years to double income between the cases is about 3.6 years.

Our findings on how to improve the inferior equilibrium in developing countries are as follows. First, it is important that the government undertake the accumulation of public health infrastructure by levying a moderate income tax on individuals. Second, it is also important for the state to arrange for the steady provision of infrastructure. By taking advantage of these policy solutions to reduce the growth rate gap, governments in developing countries can improve the low-growth state, even under multiple equilibria.

[31] See Table 2.

4. Properties of a dynamic system

4.1. Dynamic system

In this section, we summarize a dynamic system under our model and examine its stability properties. First, from the dynamic equation for physical capital and (13)–(15), our dynamic system is represented by the following four differential equations:

$$\frac{\dot{K}}{K} = (1-\tau)(1-l)^{1-\alpha}\left(\frac{K}{H}\right)^{\alpha-1} - \frac{C}{K}, \tag{23}$$

$$\frac{\dot{H}}{H} = \delta\tau(1-l)^{1-\alpha}\left(\frac{K}{H}\right)^{\alpha}, \tag{24}$$

$$\begin{aligned}\frac{\dot{C}}{C} = &\frac{\alpha}{b}(1-\tau)(1-l)^{1-\alpha}(1-\eta(1-\theta))\left(\frac{K}{H}\right)^{\alpha-1} \\ &+ \frac{1-\theta}{b}\delta\tau(1-l)^{1-\alpha}(\sigma-\eta(1-\alpha))\left(\frac{K}{H}\right)^{\alpha} \\ &+ \frac{\alpha\eta(1-\theta)}{b}\left(\frac{C}{K} - \frac{\dot{l}}{1-l}\right) - \frac{\rho}{b},\end{aligned} \tag{25}$$

$$\begin{aligned}\frac{\dot{l}}{l} = &\frac{\dot{C}}{C} - \alpha(1-\tau)(1-l)^{1-\alpha}\left(\frac{K}{H}\right)^{\alpha-1} + \alpha\left(\frac{C}{K} - \frac{\dot{l}}{1-l}\right) \\ &- \delta\tau(1-\alpha)(1-l)^{1-\alpha}\left(\frac{K}{H}\right)^{\alpha},\end{aligned} \tag{26}$$

where $\theta - \eta(1-\theta) \equiv b$.[32] In view of tractability, the present four-dimensional system can be converted to a new system (see Appendix B). As a result, these three equations also characterize the dynamics of the model.

Unfortunately, it is a rather formidable task for us to attempt further analytical investigation. To overcome this difficulty, θ is set to unity. As is well-known, under this assumption, our utility function reduces to a logarithmic form, satisfying additive separability. In our setting, $\theta = 1$ corresponds to the case in which the equilibrium is uniquely determined. It is therefore impossible to examine the dual steady-states case, even though we are especially interested in

[32] Note that (24) is identical to (14).

this case.[33] This is an issue for the future. As a result, our simplified dynamic system is as follows:

$$\frac{\dot{X}}{X} = (\alpha - 1)(1 - \tau)(1 - l)^{1-\alpha} Z^{\alpha-1} - X - \rho, \tag{27}$$

$$\frac{\dot{Z}}{Z} = (1 - l)^{1-\alpha}((1 - \tau)Z^{-1} - \delta\tau)Z^{\alpha} - X, \tag{28}$$

$$\frac{\dot{l}}{l} = \frac{(1 - l)(\delta\tau(\alpha - 1)(1 - l)^{1-\alpha} Z^{\alpha} + \alpha X - \rho)}{1 - l(1 - \alpha)}. \tag{29}$$

4.2. Transitional dynamics and local stability

To examine the local stability of the equilibrium, we linearize the reduced dynamic system (27)–(29) under the assumption $\theta = 1$, associated with the original system (23)–(26), around the steady state (SS). The steady-state values are denoted as X^*, Z^* and l^*. Consequently, we obtain the following linear system:

$$\begin{bmatrix} \dot{X}(t) \\ \dot{Z}(t) \\ \dot{l}(t) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} X(t) - X^* \\ Z(t) - Z^* \\ l(t) - l^* \end{bmatrix}, \tag{30}$$

[33] In the stability analysis, Raurich (2003) also concentrates on the case of unique equilibrium. Such simplification is common in the literature when covering the issue of multiple equilibria.

where

$$
\begin{aligned}
a_{11} &= \left.\frac{\partial \dot{X}}{\partial X}\right|_{\mathrm{SS}} = -X^* < 0, \\
a_{12} &= \left.\frac{\partial \dot{X}}{\partial Z}\right|_{\mathrm{SS}} = -(1-\alpha)X^*Z^*(X^*+\rho) < 0, \\
a_{13} &= \left.\frac{\partial \dot{X}}{\partial l}\right|_{\mathrm{SS}} = \frac{-(1-\alpha)X^*(X^*+\rho)}{1-l^*} < 0, \\
a_{21} &= \left.\frac{\partial \dot{Z}}{\partial X}\right|_{\mathrm{SS}} = -Z^* < 0, \\
a_{22} &= \left.\frac{\partial \dot{Z}}{\partial Z}\right|_{\mathrm{SS}} = -(1-\alpha)X^* - \delta\tau(1-l^*)^{1-\alpha}(Z^*)^{\alpha} < 0, \\
a_{23} &= \left.\frac{\partial \dot{Z}}{\partial l}\right|_{\mathrm{SS}} = \frac{(1-\alpha)X^*Z^*}{1-l^*} > 0, \\
a_{31} &= \left.\frac{\partial \dot{l}}{\partial X}\right|_{\mathrm{SS}} = \frac{\alpha l^*(1-l^*)}{1-l^*(1-\alpha)} > 0, \\
a_{32} &= \left.\frac{\partial \dot{l}}{\partial Z}\right|_{\mathrm{SS}} = \frac{-\alpha\delta\tau(1-\alpha)l^*(1-l^*)^{2-\alpha}(Z^*)^{\alpha-1}}{1-l^*(1-\alpha)} < 0, \\
a_{33} &= \left.\frac{\partial \dot{l}}{\partial l}\right|_{\mathrm{SS}} = \frac{(1-\alpha)l^*(\alpha X^*-\rho)}{1-l^*(1-\alpha)} > \text{or} < 0.
\end{aligned}
$$

Our 3×3 system is composed of one control variable, l, one control-like variable, X, both of which are jump variables, and one state-like variable, Z, whose initial value $Z(0)$ is predetermined.[34] To examine the local stability properties, we first confirm the sign of the trace of the Jacobian matrix, $\mathbf{J}^*$, in (30).[35] From the elements of $\mathbf{J}^*$, if $a_{33} < 0$, $\mathrm{Tr}\,\mathbf{J}^* < 0$ holds. However, since the sign of $\alpha X^* - \rho$ is ambiguous, the sign of $\mathrm{Tr}\,\mathbf{J}^*$ is not determined. So obtaining $\mathrm{Tr}\,\mathbf{J}^*$ directly, it always has a negative sign. Based on this result and the Routh-Hurwitz theorem, there are four possible cases. Following Benhabib and Perli (1994) and Chen and Lee (2007), we display the cases in Table 3.

[Insert Table 3 around here]

[34] The initial value $X(0) \equiv C(0)/K(0)$ is not predetermined, since $C(0)$ is a jump variable.

[35] The trace and determinant of $\mathbf{J}^*$ are designated as $\mathrm{Tr}\,\mathbf{J}^*$ and $\mathrm{Det}\,\mathbf{J}^*$, respectively.

The Routh-Hurwitz theorem states that the number of characteristic roots of the corresponding polynomial with positive real parts is equal to the number of variations of sign in the scheme $[-1, \mathrm{Tr}\,\mathbf{J}^*, -\mathrm{B}\,\mathbf{J}^* + \mathrm{Det}\,\mathbf{J}^*/\mathrm{Tr}\,\mathbf{J}^*, \mathrm{Det}\,\mathbf{J}^*]$, where $\mathrm{B}\,\mathbf{J}^*$ denotes the determinant of the bordered Hessian of $\mathbf{J}^*$.[36] The first three rows of Table 3 show the cases in which the number of changes in signs are less than or equal to one, so the number of characteristic roots with negative real parts is either 2 or 3. In a similar way, the fourth row of Table 3 represents the case in which the number of negative characteristic roots is 1.

In any case, to determine the corresponding scenario, we need to obtain the signs of $-\mathrm{B}\,\mathbf{J}^* + \mathrm{Det}\,\mathbf{J}^*/\mathrm{Tr}\,\mathbf{J}^*$ and $\mathrm{Det}\,\mathbf{J}^*$. Because of the ambiguity of a_{33}, it is difficult to determine the sign of $\mathrm{Det}\,\mathbf{J}^*$. Now, we employ the special assumption which satisfies $\alpha X^* = \rho$. Although this is an ad-hoc assumption, it makes sense in a way. The values $\alpha = 0.35$ and $\rho = 0.05$ that we used in the previous numerical analysis lead to $X^* = 0.143$. Since $X \equiv C/K$, this is a relatively reasonable assumption. Applying $\alpha X^* = \rho$, we have

$$\begin{aligned}\mathrm{Det}\,\mathbf{J}^* &= -\frac{\alpha\delta\tau l^*(1-\alpha)^2(1-l^*)^{1-\alpha}X^*(Z^*)^\alpha(2X^*+\rho)}{1-l^*(1-\alpha)} \\ &- \frac{\alpha(1-\alpha)l^*X^*(X^*+\rho)((1-\alpha)X^*(1+(Z^*)^2)+\delta\tau(1-l^*)^{1-\alpha}(Z^*)^\alpha)}{1-l^*(1-\alpha)} < 0.\end{aligned}$$

Since $\mathrm{Det}\,\mathbf{J}^* < 0$, the remaining cases are limited to the first and fourth row of Table 3. We summarize the current cases as follows.

Proposition 3. $-\mathrm{B}\,\mathbf{J}^* + \mathrm{Det}\,\mathbf{J}^*/\mathrm{Tr}\,\mathbf{J}^* > 0$ *if* $\mathrm{B}\,\mathbf{J}^* < 0$. *Here,* $\mathbf{J}^*$ *has only one negative characteristic root, and thus the equilibrium is locally determinate (i.e., locally saddle-path stable).*

The negative eigenvalue corresponds to a stable root and constitutes the stable trajectory converging to the unique steady state. At the same time, we arrive

[36] As is well known, the characteristic roots of $\mathbf{J}^*$ are the solutions of the following characteristic polynomial:

$$-\mu^3 + \mathrm{Tr}\,\mathbf{J}^*\mu^2 - \mathrm{B}\,\mathbf{J}^*\mu + \mathrm{Det}\,\mathbf{J}^* = 0,$$

where μ denotes the eigenvalue.

at the following.

Proposition 4. *If* $-\mathrm{B}\,\mathbf{J}^* + \mathrm{Det}\,\mathbf{J}^*/\mathrm{Tr}\,\mathbf{J}^* < 0$ *is satisfied,* $\mathbf{J}^*$ *has three negative roots. It is possible that in this case the equilibrium is locally indeterminate.*

Indeterminacy implies that there are multiple converging paths in the neighborhood of the unique steady state.

To sum up our local stability analysis, under the assumptions of $\theta = 1$ and $\alpha X^* = \rho$, we obtain the result that there are either determinate or indeterminate equilibrium path, depending on the sign of $-\mathrm{B}\,\mathbf{J}^* + \mathrm{Det}\,\mathbf{J}^*/\mathrm{Tr}\,\mathbf{J}^*$. We present a typical numerical example for reference below.

Example. Using the two assumptions and a set of standard parameters ($\alpha = 0.35$, $\tau = 0.05$, $\delta = 0.3$, $\rho = 0.05$ and $l^* = 0.6$), the sign of the bordered Hessian is obviously negative for any acceptable value of Z^*.

Therefore, it is highly likely that the equilibrium is locally saddle-path stable.

5. Concluding remarks

In this paper, on the basis of theoretical and numerical analyses, we examined how publicly provided health infrastructure can affect the long-run growth of the economy, allowing for an elastic labor supply. Infrastructure has a direct influence on production activities through the enhancement of labor productivity, which has a positive impact on the utility of agents. By extending the model to accept a relatively large intertemporal elasticity of substitution, which is consistent with the recent evidence, we show that there is the possibility of multiple equilibria, in addition to the standard case of a unique equilibrium.

The key insights of this paper cover the case of multiple equilibria. In comparison with the case of a single steady state, the multiple equilibria case produced different results in several respects. As a particularly interesting result, we find that the more an agent benefits from exogenously given infrastructure levels in their own utility, the growth rate at the high-growth steady state declines notably. The fact that a change in the welfare weight of infrastructure almost exclusively affects the high-growth equilibrium was of great interest. Moreover, we can confirm that an increase in the efficiency of infrastructure provision enhances the growth rate at the low-growth steady state. This finding implies a way out of the low-growth state in developing countries. For example, in regard to providing foreign aid, our result emphasizes the importance of creating and maintaining public health infrastructure. Further, an increase in labor supply has a greater effect on the low-growth steady state and results in an increase in the growth rate. Connecting these results to actual policy, we point to possible improvements in the unemployment problem in developing countries.

The highlight of this paper was our exploration, through numerical studies, of how development policies are effective in improving the low-growth state, taking into account the feasibility of the policies. From the viewpoint of macro-stabilization policy, we defined good policies as reducing the growth rate gap between the high- and low-growth equilibrium, and conducted an analysis in this direction. As a result, two useful policies emerged. First, when the government collects taxes at an appropriate tax rate and expends that revenue on the provision of infrastructure, the disparity between the two steady states can be reduced considerably. Second, creating a more efficient process for the provision of infrastructure was found to play a crucial role in improving the inferior equilibrium. These results offer important insights for economic development and macroeconomic policies in developing countries.

Also, we should note the properties of our dynamic system. Since the original system is rather complex, we were forced to attempt an analysis upon accepting some specific assumptions. We showed that for the local stability of the (unique) equilibrium, they are either saddle-path stable or indeterminate. For the considered scenarios, numerical exploration proved that the case gives rise to saddle-path stability.

Finally, for further investigation, we provided an essential extension of the research findings given in this paper. An increase in the difficulty of our challenge is inevitable, but it is important to enrich the analysis by introducing individual health capital, in addition to the health infrastructure which is exogenously supplied by the government.

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Appendix A. Derivation of (13)

Log-differentiating (12), we have

$$\frac{\dot{l}}{l} = \frac{\dot{C}}{C} - \frac{\dot{w}}{w} - \frac{\dot{H}}{H}.$$

Here, applying the expression $\dot{w}/w = \alpha(\dot{K}/K) + \alpha(\dot{l}/1-l) - \alpha(\dot{H}/H)$ to the above yields:

$$\frac{\dot{l}}{l} = \frac{\dot{C}}{C} - \alpha\frac{\dot{K}}{K} - \alpha\frac{\dot{l}}{1-l} - (1-\alpha)\frac{\dot{H}}{H}.$$

Finally, substituting the expression $\dot{K}/K = (1-\tau)(1-l)^{1-\alpha}(K/H)^{\alpha-1} - C/K$ into the equation above, we obtain (13) in the text.

Appendix B. A condensed dynamic system

Using $X \equiv C/K$, $Z \equiv K/H$ and (23)–(26), we have

$$\begin{aligned}
\frac{\dot{X}}{X} &= \frac{\alpha(1-\tau)(1-l)^{1-\alpha}(1-\eta(1-\theta))Z^{\alpha-1}}{b} \\
&\quad + \frac{\delta\tau(1-\theta)(1-l)^{1-\alpha}(\sigma-\eta(1-\alpha))Z^{\alpha}+\alpha\eta(1-\theta)X-\rho}{b} \\
&\quad - \frac{\alpha^2\eta(1-\tau)(1-\theta)^2(1-l)^{1-\alpha}lZ^{\alpha-1}}{b((1-l)b+\alpha\theta l)} \\
&\quad - \frac{\alpha\eta\delta\tau(1-\theta)(\sigma(1-\theta)-\theta(1-\alpha))(1-l)^{1-\alpha}lZ^{\alpha}}{b((1-l)b+\alpha\theta l)} \\
&\quad - \frac{\alpha\eta(1-\theta)l(\alpha\theta X-\rho)}{b((1-l)b+\alpha\theta l)} - (1-\tau)(1-l)^{1-\alpha}Z^{\alpha-1} - X, \\
\frac{\dot{Z}}{Z} &= (1-\tau)(1-l)^{1-\alpha}Z^{\alpha-1} - \delta\tau(1-l)^{1-\alpha}Z^{\alpha} - X
\end{aligned}$$

and

$$\begin{aligned}
\frac{\dot{l}}{l} &= \frac{\alpha(1-\tau)(1-\theta)(1-l)^{2-\alpha}Z^{\alpha-1}}{(1-l)b+\alpha\theta l} \\
&\quad + \frac{\delta\tau(1-l)^{2-\alpha}(\sigma(1-\theta)-\theta(1-\alpha))Z^{\alpha}+(1-l)(\alpha\theta X-\rho)}{(1-l)b+\alpha\theta l}.
\end{aligned}$$

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Table 1

Benchmark parameters (single steady state).

α	τ	δ	σ	θ	ρ	l^*
0.35	0.05	0.3	0.8	1.2	0.05	0.6

Table 2

Benchmark parameters (dual steady states).

α	τ	δ	σ	θ	ρ	l^*
0.35	0.05	0.65	3.7	0.35	0.1	0.75

Table 3

Equilibrium properties of the case with one state and two control variables.

-1	Tr $\mathbf{J}^*$	$-$B $\mathbf{J}^* +$ Det $\mathbf{J}^*$/Tr $\mathbf{J}^*$	Det $\mathbf{J}^*$	Num. of negative roots	Path toward SS
$-$	$-$	$-$	$-$	3	Indeterminate
$-$	$-$	$-$	$+$	2	Indeterminate
$-$	$-$	$+$	$+$	2	Indeterminate
$-$	$-$	$+$	$-$	1	Determinate

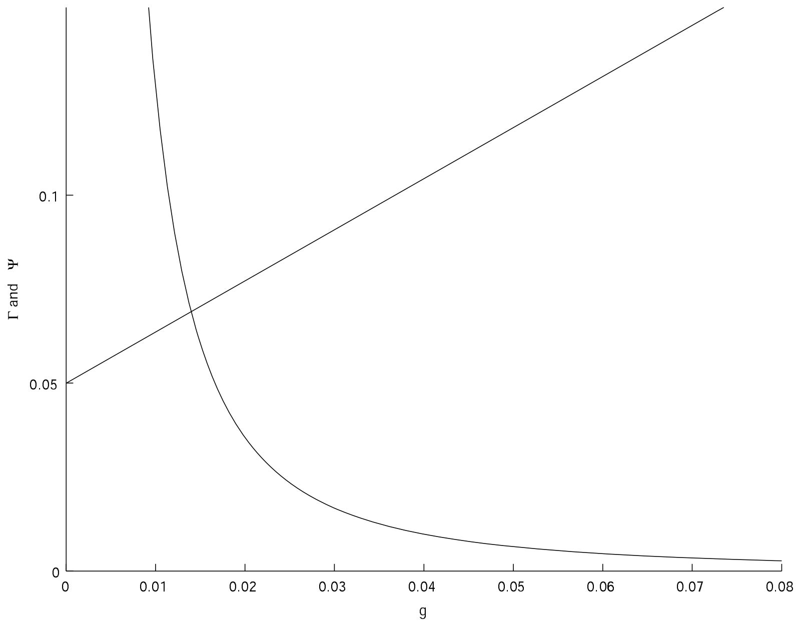

Figure 1: Benchmark case (single steady state)

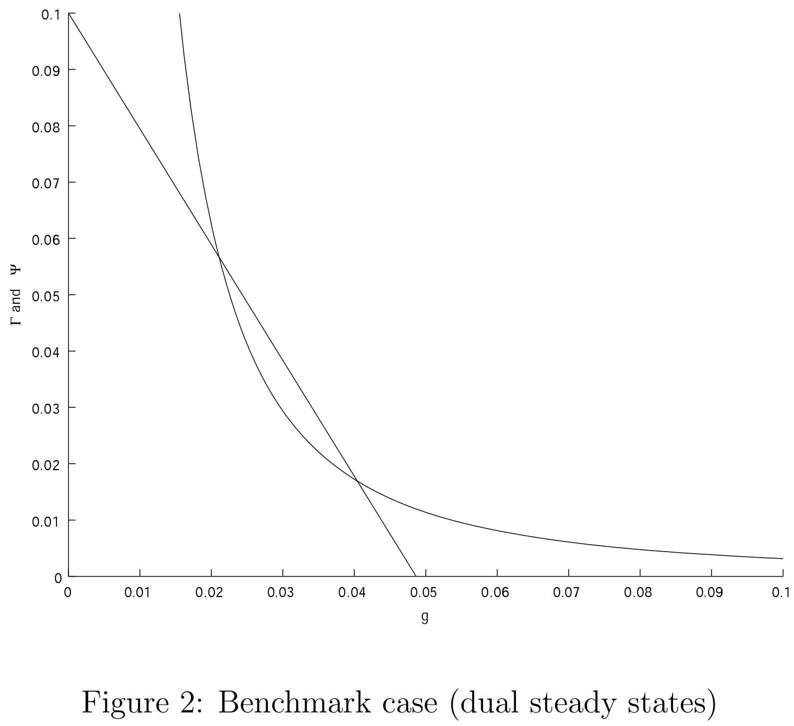

Figure 2: Benchmark case (dual steady states)

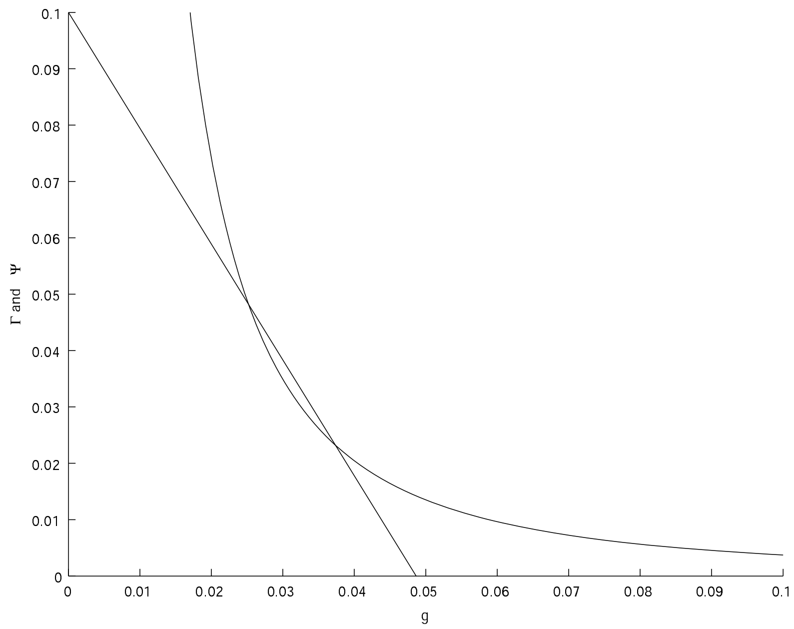

Figure 3: A change in τ from 0.05 to 0.055 (dual steady states)

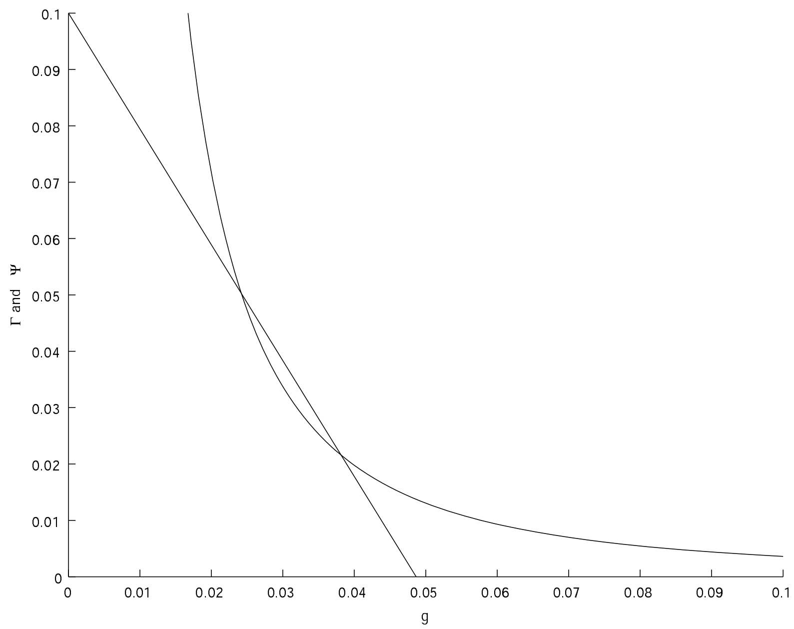

Figure 4: A change in δ from 0.65 to 0.7 (dual steady states)

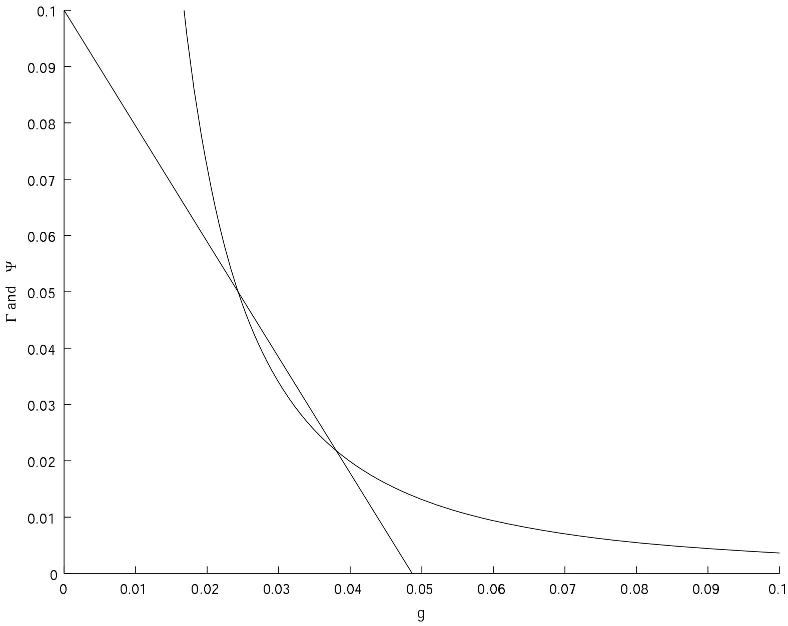

Figure 5: A change in l^* from 0.75 to 0.73 (dual steady states)

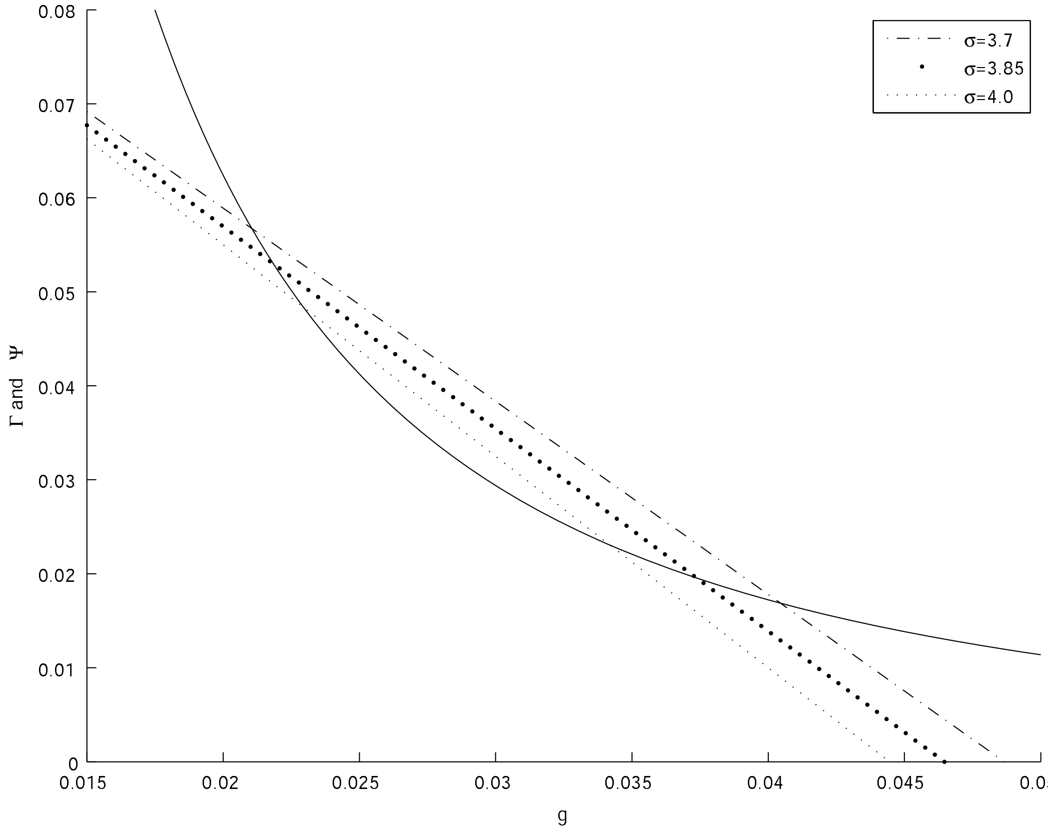

Figure 6: A change in σ from 3.7 to 3.85 and 4.0 (dual steady states)

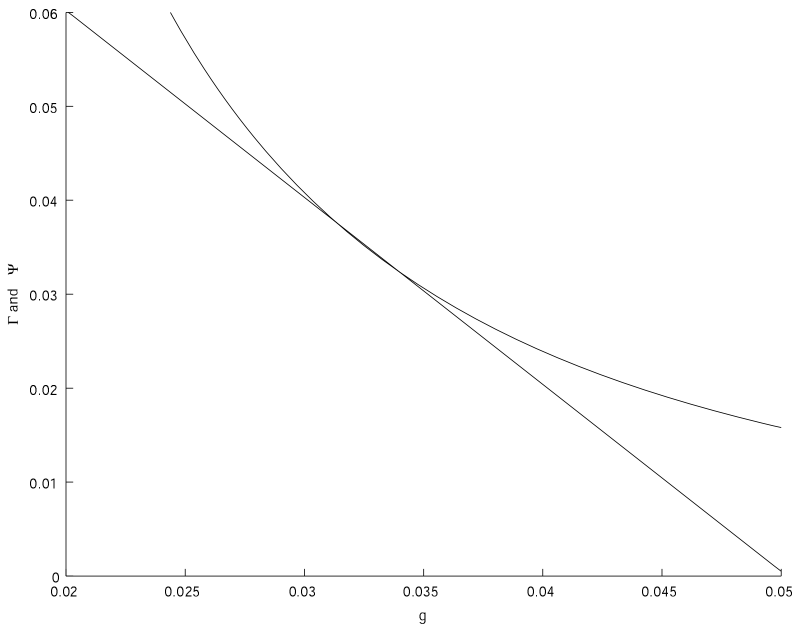

Figure 7: Improving the low-growth state (tax policy impact)

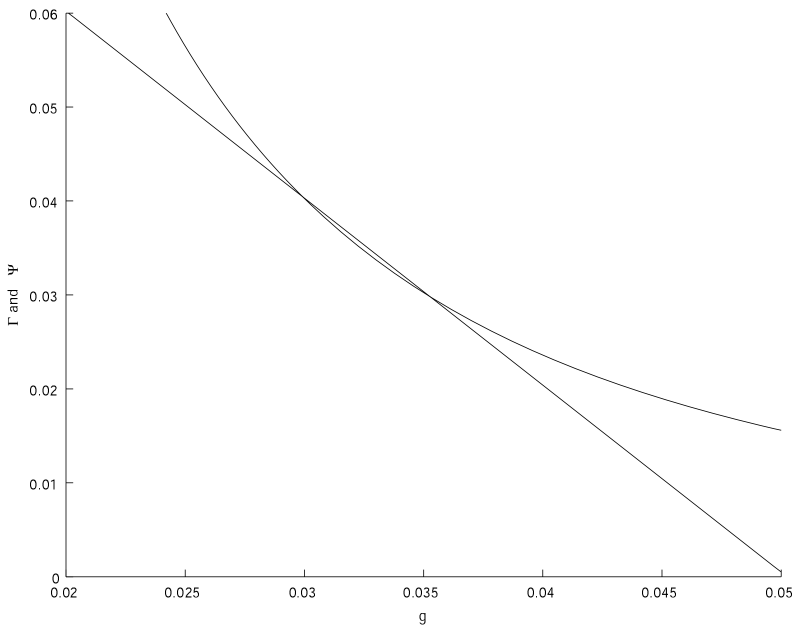

Figure 8: Improving the low-growth state (public health policy impact)