Monetary Policy, Financial Uncertainty, and Secular Stagnation

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Abstract

A monetary policy framework describing how to cope with a financial crisis might alleviate a recession, but might also result in subsequent secular stagnation. Based on an empirical New Keynesian model with financial uncertainty, this study investigates how monetary policy can avoid sluggish economic recovery in response to financial shocks. The results show that a protracted sluggish response of an output gap is triggered by inflation targeting, without taking into account interest rate variations. In such a policy, the uncertainty causes additional sluggish behavior after the sharp reduction in the output gap. In contrast, in a speed limit policy, the output gap recovers rapidly, regardless of the central bank’s approach to interest rate variations, and the uncertainty mitigates reductions in the output gap. Finally, the results are robust to checks under several alternative settings.

Keywords: Monetary policy; Financial uncertainty; Secular stagnation; Inflation targeting; Speed limit policy

JEL classification: E32; E62

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1 Introduction

To counteract the aftermath of the global financial crisis and the Great Recession in the late 2000s, the Federal Reserve in the United States promptly cut the federal funds rate to nearly zero, and subsequently undertook unconventional monetary policies. However, despite the prompt and bold policy measures, the United States has experienced secular stagnation since the crisis. As shown in Figure 1, the U.S. economy will take roughly 10 years to fill the large negative output gap stemming from the crisis.\(^1\) In recent influential works, Summers (2014, 2015) argues that the negative natural rate of interest and the resultant insufficient effects of monetary policies under a zero nominal lower bound are related to the recent secular stagnation.

[Insert Figure 1 around here]

Importantly, slow recoveries from financial crises are not uncommon. According to Reinhart and Rogoff (2014), who studied approximately 100 systemic banking crises since the latter half of the 19th century, protracted sluggish recovery is observed in most of the crises. It is highly unlikely that all historical recoveries from financial crises can be explained adequately by a negative natural rate of interest and zero lower bound effects.

In this study, we provide several complementary explanations for the origins of secular stagnation. That is, rather than focusing on the inadequacy of monetary policy with a zero nominal lower bound, we focus on side effects and how to address a financial crisis and subsequent recessions. We argue that while recessions are regarded as temporary fluctuations in modern macroeconomics, they can be prolonged, depending on the way in which a central bank manages the situation.

It is possible that a protracted sluggish output gap response to a financial shock is highly relevant to policymakers’ targets. The literature on optimal monetary policy in dynamic stochastic general equilibrium frameworks provides several candidates as appropriate monetary policy objectives. These include inflation targeting (Svensson, 1997) and speed limit policies to stabilize inflation and changes in the output gap (Walsh, 2003). However, little is known about the difference in performance between inflation targeting and speed limit policies when financial shocks occur. Furthermore, whether or not policymakers take changes in interest rates into account is crucial to monetary policy behavior. On the one hand, in a financial crisis, it is possible to justify stabilizing inflation and the output gap, without worrying about variations in policy interest rates. On the other

\(^1\)Note that the potential output has been revised downward since the Great Recession, as pointed out by Summers (2014, 2016). While beyond the scope of this study, an output gap contraction is related to a decline in potential output and a recovery in demand.
hand, policy interest rates need to be smoothed, with moderate cuts, especially to avoid facing a zero lower bound.

In addition, when evaluating the macroeconomic dynamics of financial shocks, it seems essential to consider the effects of uncertainty. Amid a financial crisis, policymakers recognize that credit and financial markets are disrupted and private economic activities change structurally. However, they tend to expect that the crisis will soon be over, market disruption will end, and the economy will return to normal. Importantly, history shows that crises can recur (e.g., Reinhart and Rogoff, 2014). Hence, it is highly likely that normal and crisis periods will be repeated, and central banks reflect such repeated regime-switching in their behavior.

Motivated by the aforementioned studies, this study has two objectives, aimed at enriching our knowledge on secular stagnation. First, we attempt to clarify which targets policymakers should adopt to prevent an output gap recovery from becoming protracted in response to a financial shock. As such, we compare inflation targeting and speed limit policies, both with and without being concerned about interest rate variations. Second, while comparing these monetary policy objectives, we attempt to quantify the effects of financial uncertainty. As a result, we aim to identify which objectives show robust performance in the face of uncertainty.

To achieve these objectives, we employ the Markov jump-linear-quadratic approach developed by Svensson and Williams (2007), among others. This framework allows us to examine how key macroeconomic variables fluctuate under uncertainty for various central bank objectives. Specifically, our analysis builds on the work of Williams (2012), who applies the same approach to the effects of financial uncertainty on optimal monetary policies. However, the original study focuses only on inflation targeting with interest rate smoothing. Following Williams (2012), we use an empirical New Keynesian model with financial uncertainty to examine which central bank objectives can lead to a rapid recovery from a financial crisis.

Our main findings are summarized as follows. In the case of inflation targeting, we find that a negative output gap response to a financial shock becomes protracted unless the central bank strives to stabilize interest rate variations. In such a policy, uncertainty causes sluggish behavior after the sharp reduction in the output gap. Unlike the case of inflation targeting, a recovery can be attained within a relatively short period using speed limit policies. Moreover, in a speed limit policy, the rapid recovery is insensitive to the central bank’s approach to interest rate variations, and reductions in the output gap are mitigated by the uncertainty. In other words, a speed limit policy outperforms inflation targeting by avoiding secular stagnation after a financial crisis. In summary, targeting frameworks and financial uncertainty have greater implications for secular stagnation than is currently believed to be the case.
1.1 Related literature

There is increasing concern about the slow recovery following the Great Recession. Recent studies on secular stagnation have provided several explanations for this slow recovery. One prominent explanation for the cause of such stagnation is a negative natural rate of interest and the limiting effect on monetary policy of a zero lower bound, as argued by Summers (2014, 2015). While Summers’ argument is not based on rigorous theoretical models, Barsky et al. (2014) discuss the relationship between the natural rate of interest rate and monetary policy using a DSGE model. Eggertsson et al. (2017) provide a quantitative life cycle model to characterize secular stagnation. In an empirical study, Albonico et al. (2017) find that the zero lower bound effect has caused jobless recovery since the Great Recession.

Moreover, there is wide-ranging debate among researchers over the cause of secular stagnation. For example, Taylor (2014) argues that it is the result of various policies, such as regulatory policies, bailouts, discretionary fiscal policy, and monetary policy. Lo and Rogoff (2015) discuss the issue from several viewpoints, including innovation, demographics, policy uncertainty, and debt overhang. Apergis (2017) explores 127 global economies and presents evidence in favor of macroprudential policies in order for monetary policy to be effective. From a theoretical perspective, Fajgelbaum et al. (2017) demonstrate that an endogenous uncertainty mechanism can prolong recessions and account for the features of the Great Recession. Our study examines how a monetary policy framework can be used to cope with financial shocks and the effects of financial uncertainty.

In terms of the modeling framework, as already mentioned, this study is closest to that of Williams (2012), who finds that the effect of financial uncertainty is negligible. However, he only analyzes a parameter setting of inflation targeting with interest rate smoothing, disregarding speed limit policies, which outperform other monetary policy frameworks in some settings (e.g., Walsh, 2003; Yetman, 2006; Blake, 2012). Furthermore, the recent renewed interest in secular stagnation is not focused. Hence, our research questions have yet to be answered in the literature.

There exists a strand of literature that investigates monetary policy under uncertainty. Here, relevant works include those of Kimura and Kurozumi (2007), Tillmann (2009), and Kurozumi (2010). Many studies deal with the uncertainty by using Markov-switching models (e.g., Zampolli, 2006; Davig and Leeper, 2007; Farmer et al., 2009, 2011). Our investigation utilizes the Markov jump-linear-quadratic approach to solve the model. The

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2See also Benigno and Fornaro (forthcoming), who present a Keynesian growth model in which a stagnation trap arises and full employment cannot be attained by monetary policy.

3Albonico et al. (2017) also analyze the role of fiscal policies in the Great Recession, focusing on Non-Ricardian households, and find that fiscal policies have little effect.

4Caglayan et al. (2017) show that financial deepening contributes to more effective monetary policy in recessions.
approach was initially proposed by Svensson and Williams (2007), and has subsequently been applied to modeling various types of uncertainty (e.g., Svensson and Williams, 2008; Williams, 2012; Flaminia and Milas, 2015).

2 The analytical framework

In this section we briefly outline our analytical framework, building on the work of Williams (2012), with additional consideration of monetary policy targets.

2.1 The model

Consider a New Keynesian framework with financial uncertainty, as in Williams (2012), in which there are private sectors and a central bank. The model is based on a combination of the studies of Lindé (2005) and Ćurdić and Woodford (2009). In addition, we extend the standard New Keynesian model to include credit market frictions.

To represent financial uncertainty, we suppose two regimes. Here, $j_t$ is a dummy variable for period $t$ that takes the value one in a normal regime and the value two in a financial crisis regime. Furthermore, regime $j_t$ is assumed to follow a Markov process with the following $2 \times 2$ transition matrix:

$$
P = \begin{bmatrix}
    \text{Prob}(j_{t+1} = 1|j_t = 1) & 1 - \text{Prob}(j_{t+1} = 1|j_t = 1) \\
    1 - \text{Prob}(j_{t+1} = 2|j_t = 2) & \text{Prob}(j_{t+1} = 2|j_t = 2)
\end{bmatrix}.
$$

(1)

With regard to the dynamics of the private sector, a New Keynesian Phillips curve and a New Keynesian IS equation are used. These include credit market frictions, backward-components of the inflation rate $\pi_t$, and the output gap $y_t$:

$$
\pi_t = \omega_f j_t E_t \pi_{t+1} + (1 - \omega_f j_t) \pi_{t-1} + \gamma_{j_t} y_t + \xi_{j_t} \Omega_t + c_{j_t} \varepsilon_{\pi t},
$$

(2)

$$
y_t = \beta_f j_t E_t y_{t+1} + (1 - \beta_f j_t) [\beta_{y_{j_t}} y_{t-1} + (1 - \beta_{y_{j_t}}) y_{t-2}] - \beta_{y_{j_t}} (i_t - E_t \pi_{t+1}) + \theta_{j_t} \Omega_t + \phi_{j_t} \omega_t + c_{y_{j_t}} \varepsilon_{yt},
$$

(3)

where $E_t$ is the mathematical conditional expectation based on information available in period $t$, $i_t$ is the nominal interest rate, and $\varepsilon_{\pi t}$ and $\varepsilon_{yt}$ are mutually uncorrelated standard normal random shocks, such that

$$
\begin{bmatrix}
    \varepsilon_{\pi t} \\
    \varepsilon_{yt}
\end{bmatrix} \sim \text{i.i.d.} N(0_{2 \times 1}, I_2).
$$

(4)

There are two key factors in this model: the marginal utility gap $\Omega_t$ and the spread
interest rate $\omega_t$ between borrowers and savers. Note that the model of the normal regime is nested in the model of the financial crisis regime, $\xi_1 = \theta_1 = \phi_1 = 0$. In other words, in a normal regime, the fourth term on the right-hand side of (2) and the fourth and fifth terms on the right-hand side of (3) are eliminated. We assume that the dynamics of the marginal utility gap $\Omega_t$ are endogenous, and that the spread rate $\omega_t$ exogenously follows an AR(1) stochastic process:

$$
\Omega_t = \delta E_t \Omega_{t+1} + \omega_t, \quad (5)
$$

$$
\omega_t = \rho \omega_{t-1} + \epsilon \omega_{t}, \quad (6)
$$

where $\epsilon_{\omega t}$ is the spread shock, which is mutually uncorrelated with $\epsilon_{\pi t}$ and $\epsilon_{yt}$, and $\epsilon_{\omega t} \sim \text{i.i.d.} N(0, 1)$.

We consider two frameworks of central bank loss functions, including the penalties associated with some types of interest rate variations. The first is inflation targeting, as in Williams (2012), and the second is the speed limit policy, as in Walsh (2003). The period loss function for inflation targeting is

$$
L^{IT} (\pi_t, y_t, i_t, i_{t-1}, j_t) = \pi_t^2 + \lambda y_t^2 + \nu (i_t - i_{t-1})^2 + \psi_i i_t^2, \quad (7)
$$

and that of the speed limit policy is

$$
L^{SLP} (\pi_t, y_t, y_{t-1}, i_t, i_{t-1}, j_t) = \pi_t^2 + \lambda (y_t - y_{t-1})^2 + \nu (i_t - i_{t-1})^2 + \psi_j i_t^2. \quad (8)
$$

In both frameworks, the third term represents the penalty associated with interest rate smoothing and the fourth term represents that of interest rate volatility. Following Williams (2012), the latter penalty is allowed to change across regimes in order to address the zero lower bound issue (i.e., $\psi_1 < \psi_2$).\(^5\) With respect to the instrument $i_t$, the central bank is assumed to minimize the intertemporal loss function in the case of inflation targeting:

$$
E_t \sum_{\tau=0}^{\infty} \beta^\tau L^{IT} (\pi_{t+\tau}, y_{t+\tau}, i_{t+\tau}, i_{t+\tau-1}, j_{t+\tau}), \quad (9)
$$

and in the speed limit policy case:

$$
E_t \sum_{\tau=0}^{\infty} \beta^\tau L^{SLP} (\pi_{t+\tau}, y_{t+\tau}, y_{t+\tau-1}, i_{t+\tau}, i_{t+\tau-1}, j_{t+\tau}), \quad (10)
$$

\(^5\)Unfortunately, as found in Williams (2012) as well, introducing the effect of a zero nominal lower bound is difficult in our model. In addition, while other types of asymmetry in the central bank’s preferences are considered in the literature (e.g., Chesang and Naraidoo, 2016), they are unlikely to be incorporated into the present framework.
subject to (2)–(6), where $\beta$ is the discount factor of the central bank.

## 2.2 Markov jump-linear-quadratic approach

As in Williams (2012), we solve the above dynamic optimization problem utilizing the Markov jump-linear-quadratic approach proposed by Svensson and Williams (2007), as follows.

The dynamic system of (2)–(6) can be rewritten more compactly in matrix notation as

\[ x_{t+1} = A_{11j_{t+1}} x_t + A_{12} z_t + b_1 + C_{j_{t+1}} e_{t+1}, \quad (11) \]

\[ E_t H_{j_{t+1}} z_{t+1} = A_{21j_t} x_t + A_{22j_t} z_t + b_{2j_t}, \quad (12) \]

where $x_t$ and $z_t$ are stacked vectors of predetermined variables and forward-looking variables, respectively,

\[ x_t = \begin{bmatrix} x_{t-1}, y_{t-1}, \ldots \end{bmatrix}_t, \quad (13) \]

\[ z_t = \begin{bmatrix} x_t, y_t, \ldots \end{bmatrix}_t, \quad (14) \]

and $e_t$ is a three-dimensional vector of shocks

\[ \begin{bmatrix} e_{xt} \\ e_{yt} \\ e_{\omega t} \end{bmatrix} \sim \text{i.i.d.} \mathcal{N}(0_{3x1}, I_3). \quad (15) \]

Note that the coefficient matrices $A_{11j_t}$, $C_{j_t}$, $H_{j_t}$, $A_{21j_t}$, and $A_{22j_t}$, and the coefficient vectors $b_{2j_t}$ depend on the regimes and are random. Furthermore, the coefficient matrix $A_{12}$ and the coefficient vector $b_1$ are deterministic.\(^6\) The matrix $A_{22j_t}$ is required to be non-singular for $\forall j_t$ to ensure that forward-looking variables can be calculated, such that

\[ z_t = A_{22j_t}^{-1} (E_t H_{j_{t+1}} z_{t+1} - A_{21j_t} x_t - b_{2j_t}). \quad (16) \]

Turning now to the central bank’s loss function, let $y_t$ denote a vector of target variables, defined as:

\[ y_t \equiv D \begin{bmatrix} x_t \\ z_t \\ e_t \end{bmatrix}, \quad (17) \]

\(^6\)See Appendix A for the explicit representation of the coefficient matrices and vectors.
where $\mathbf{D}$ is a matrix that selects target variables within predetermined, forward-looking, and instrument variables. In addition, let $\mathbf{A}_t$ denote a matrix that represents the central bank’s weight among the target variables. Thus, under inflation targeting or a speed limit policy, the period loss function can be expressed as

$$L^{MP}(x_t, z_t, i_t, j_t) = y_t' \Lambda_j y_t = \begin{bmatrix} x_t \\ z_t \\ i_t \end{bmatrix}' \mathbf{D}' \mathbf{A}_t \mathbf{D} \begin{bmatrix} x_t \\ z_t \\ i_t \end{bmatrix} = \mathbf{x}_t' \mathbf{W}_{j_t} \mathbf{z}_t,$$  \hspace{1cm} (18)$$

where $MP \in \{IT, SLP\}$ and $\mathbf{W}_{j_t} = \mathbf{D}' \mathbf{A}_t \mathbf{D}$ (see Appendix B). We can now regard the dynamic optimization problem as minimizing (9) or (10) with respect to $i_t$, subject to (11), (16), and (18). Owing to the presence of forward-looking variables, the dynamic optimization problem depending on $E_t \mathbf{H}_{j_{t+1}} \mathbf{z}_{t+1}$ cannot be solved recursively. To obtain optimal policies, Svensson and Williams (2007) rewrite the problem to resolve this difficulty using the recursive saddle point method (see Appendix C).

Before proceeding to the results, we explain our informational assumptions. We assume “unobservable regimes,” as in Svensson and Williams (2007): both the central bank and private sector know the probability distribution of $\varepsilon_t$, the transition matrix $\mathbf{P}$, and the values of the coefficient matrices in both regime, but they cannot observe the modes $j_t$. Specifically, their information set includes their subjective distribution of regimes $\mathbf{p}_t = [p_{1t}, p_{2t}]'$, rather than regimes $j_t$, and is given exogenously pursuant to

$$\mathbf{p}_{t+\tau} = (\mathbf{P})^\tau \mathbf{p}_t, \text{ for } \forall \tau \geq 0,$$  \hspace{1cm} (19)$$

which means they unlearn observations and do not update their subjective distribution of regimes.

### 3 Numerical analysis

#### 3.1 The baseline parameter settings

As a preliminary step to solving the model numerically, we first set the parameters. The values of the private sector’s behavior come from estimations by Williams (2012, Table 1). In addition, following his estimation, the transition matrix is set to

$$\mathbf{P} = \begin{bmatrix} 0.9961 & 0.0039 \\ 0.0352 & 0.9648 \end{bmatrix}.$$
The data on the U.S. economy is approximately that from the 1980s to the early 2010s, covering the financial crisis and Great Recession. Note that the data are quarterly observations and that the model’s period is set to quarters. When estimating these parameters, Williams (2012) assumes the Taylor rule in the normal regime and the variant rule, including the credit spread, in crisis regime, as in Cúrdia and Woodford (2010).

While our focus is the monetary policy framework, the central bank’s discount factor and preference for output gap variations is similar to his setting as a benchmark. That is, from now on, unless otherwise noted, \( \beta = 1 \) and \( \lambda = 0.5 \).

### 3.2 Impulse-response analysis

Having set the parameters, excluding some of the central bank’s preferences, we can numerically analyze the optimal policy and the impulse responses to financial shocks.\(^7\) We perform 10,000 simulations of 40 periods, corresponding to 10 years. The simulated period corresponds roughly to the duration of the negative output gap in U.S. economy after the financial crisis, as shown in Figure 1. Our focus is on financial shocks amid the crisis period, so the Markov chain is initialized in the crisis regime. For the purpose of evaluating the uncertainty effect, we also report the impulse responses on the constant-coefficient model, in which the economy is assumed to remain permanently in the crisis regime.

For inflation targeting, Figure 2 displays the impulse responses of inflation, the output gap, and the interest rate to an interest rate spread shock, with various settings of the central bank’s preferences for interest rate variations: (A) naive inflation targeting \((\nu = \psi_1 = \psi_2 = 0)\); (B) inflation targeting with interest rate smoothing \((\nu = 0.5 \text{ and } \psi_1 = \psi_2 = 0)\); (C) inflation targeting with a penalty on interest rate volatility \((\nu = 0, \psi_1 = 0.1, \text{ and } \psi_2 = 0.125)\); and (D) inflation targeting with a penalty on interest rate volatility, \((\nu = 0, \psi_1 = 0.7, \text{ and } \psi_2 = 0.875)\). In Panels C and D, following Williams (2012), we specify that \( \psi_2 \) is 25% larger than \( \psi_1 \) in order to reflect an additional penalty for avoiding the zero lower bound. The solid lines indicate the median responses, and the dotted lines indicate the 90% probability bands. Dashed lines show the responses for the constant-coefficient model.

\(^7\)In our calculation, we used programs developed by Svensson and Williams (2007), which can be found on Noah Williams’s website (https://www.ssc.wisc.edu/~nwilliam/DFT_programs.htm).
imposes the penalty on interest rate variations (Panels B, C, and D). Most of the mass of the distribution of the output gap responses in Panels B, C, and D lies close to zero after roughly 20 quarters (5 years), whereas the constant-coefficient responses are more sluggish, especially in the case of interest rate smoothing (Panel B). This is the result of the relatively mild responses of interest rates.

In contrast, in Panel A (naive inflation targeting), we find that the median response of the output gap exhibits a very sluggish recovery after the big negative spike, requiring more than 40 quarters (10 years) to be restored to its former level. In the constant-coefficient case, whereas the initial negative spike is alleviated, the output gap recovery is also sluggish. It is obvious that a slow recovery results from a responsive (hawkish) monetary policy causing an extremely rapid decline in interest rates. Somewhat interestingly, this implies that it is probable that secular stagnation arises if a central bank intends to cut interest rates, without hesitating to change the interest rate when a financial crisis occurs.

Focusing on the effects of financial uncertainty, Figure 3 plots the median impulse responses of the output gap to an interest rate spread shock, calculated as deviations from the constant model response. Panel A of this figure shows the results of inflation targeting for various $\nu$, ranging from zero to one, when $\psi_1 = \psi_2 = 0$. Excluding the case where $\nu$ is almost or exactly zero, the uncertainty effects are nearly independent of $\nu$. The uncertainty leads to the output gap recovering more quickly in the vicinity of 10 quarters after the shock. Panel B shows the results for various $\psi_1$, ranging from zero to one, when $\psi_2$ is 25% larger than $\psi_1$ and $\nu = 0$. On the whole, the uncertainty effects on the output gap are small throughout the entire period, with the obvious exception of the case where $\psi_1$ is minimal. To test the role of the asymmetric assumption in the penalty on interest rate volatility, Panel C explores the case where $\psi_2$ is equal to $\psi_1$ and $\nu = 0$. Here, we find that the results of Panel C are mirrored by those of Panel B, suggesting that the asymmetric assumption is not relevant to the uncertainty effects.

[Insert Figure 3 around here]

Turning next to the speed limit policy, once again, Figure 4 shows the impulse responses of inflation, the output gap, and the interest rate to an interest rate spread shock. First, and most importantly, the output gap returns to its original level within roughly 20 quarters (5 years), regardless of the presence of the penalty on any interest rate variation. This holds with or without the uncertainty, indicating that the speed limit policy is unlikely to cause secular stagnation. It turns out that the uncertainty tends to buffer the depth of the recession. Compared with inflation targeting, the speed limit policy appears to cause the output gap to overshoot the original level in the long-run.
Once again, in the case of the speed limit policy, Figure 5 plots the median impulse responses of the output gap to an interest rate spread shock. These responses are measured in terms of deviations from constant model response. As shown in Panel A, for $\forall \nu \in [0, 1]$, the uncertainty boosts the output gap from 0 to 10 quarters when $\psi_1 = \psi_2 = 0$. The largest degree is confirmed when $\nu = 0$, which corresponds to Panel A of Figure 4. In Panels B and C, we find a relatively small impact of the uncertainty when the central bank considers interest rate volatility. We also find a high degree of similarity in the impulse responses between Panels B and C, which means the uncertainty effects are not affected by the central bank’s asymmetric behavior toward interest rate volatility.

3.3 Further discussion and robustness

Except for some of the central bank’s preference parameters, we have so far assumed the setting in Williams (2012). However, the qualitative features observed thus far might depend on the numerical values of the parameters. In this subsection, we conduct several sensitivity analyses in order to check the robustness. In particular, we are interested in the sensitivity of the central bank’s preference parameters, because these are open to debate as they are not estimated using real data.

First, in terms of the secular stagnation, the central bank’s discount factor seems relevant. While, thus far, we have assumed $\beta = 1$, the output gap dynamics of our variables might change if the central bank discounts the future (e.g., Paez-Farrell, 2012). If the central bank discounts the period loss function in the future, we expect the possibility that a future presence of the output gap is relatively permissible.

To formally test this hypothesis, Figure 6 explores the sensitivity of the discount factor. Panel A shows the output gap response results of inflation targeting when $\beta = 0.9$ ($\nu = \psi_1 = \psi_2 = 0$), together with the benchmark results shown in Panel A of Figure 2. Panel B shows similar results for the speed limit policy, together with the benchmark results shown in Panel A of Figure 4. In the sensitivity analysis, a comparison of the results reveals that if the central bank discounts the future, then the output gap recovers more sluggishly in the case of inflation targeting. In contrast, such differences are not evident in the case of the speed limit policy. Instead, the speed limit policy overshoots the output gap after about 20 quarters by more than the benchmark results. When the penalties on interest rate variations are imposed, the results are similar to those in the absence of the discount.
Second, the relative penalty on the output gap, \( \lambda \), could be relevant. Imposing larger or smaller values of \( \lambda \) yields similar results to our benchmark case, where \( \lambda = 0.5 \), at least qualitatively.

Third, a possible extension to the monetary policy framework would be to add a penalty on the interest spread to (7) and (8):

\[
\hat{L}^{IT}(\pi_t, y_t, \omega_t, i_t, i_{t-1}, j_t) \equiv \pi_t^2 + \lambda y_t^2 + \zeta_j \omega_t^2 + \nu(i_t - i_{t-1})^2 + \psi j_i i_t^2,
\]

\[
\hat{L}^{SLP}(\pi_t, y_t, y_{t-1}, i_t, i_{t-1}, j_t) \equiv \pi_t^2 + \lambda(y_t - y_{t-1})^2 + \zeta_j \omega_t^2 + \nu(i_t - i_{t-1})^2 + \psi j_i i_t^2.
\]

This extension is motivated by the estimation framework of Williams (2012), who incorporates the interest spread into an extended Taylor rule in the crisis regime when estimating the parameters. However, the impulse responses (not reported) are very similar to those without the penalty on the interest spread. Incidentally, adding a penalty on the marginal utility gap \( \Omega_t \) also yields very similar results.

Furthermore, we may ask how sensitive the results are to alternative values of the transition matrix \( P \). It is conceivable that financial crises are not as common as that indicated in the estimation by Williams (2012). While he limits the sample period, owing to data availability, the economy will remain in a normal regime for longer and in a crisis regime for less time if the period is extended. To address this issue, we recalculate the impulse responses under alternative transition matrices, such as

\[
P = \begin{bmatrix} 0.999 & 0.001 \\ 0.05 & 0.95 \end{bmatrix},
\]

in which the normal regime is absorbing to accommodate the belief that the crisis regime is unlikely to return, and vice versa. These exercises yield very similar median responses, although some tails of the distribution become somewhat longer.

Finally, from amassed empirical evidence, we might infer that, in addition to \( \psi \) and \( \zeta \), \( \lambda \) and \( \nu \) are also state-dependent (e.g., Clarida et al., 2000; Bianchi, 2012). Nonetheless, the main features of the above results are not sensitive to the introduction of the asymmetry of \( \lambda \) and \( \nu \) across the regimes.

4 Conclusion

Despite the responsive monetary policy subsequent to the financial crisis in the late 2000s, the Great Recession was followed by an extremely slow recovery. This study demonstrates that the degree to which the output gap recovers quickly from stagnation can be traced to the monetary policy adopted in response to the financial shock. We also show that
financial uncertainty can be the origin of secular stagnation.

We first establish that a sluggish output gap response is explained by pure inflation targeting, in the sense that the central bank is not concerned about interest rate variations. The sluggishness of the simulated path is shown to be stronger when financial uncertainty exists. This result is fairly consistent with the actual experience observed in the U.S. economy. In contrast, even in the absence of interest rate variation, speed limit policies are shown to deliver a more desirable outcome in terms of the output gap recovery. Moreover, in speed limit policies, financial uncertainty exerts an upward force on the output gap. Regardless of the financial uncertainty, the better performance of the speed limit policies is robust to various parameter settings, including the case when the central bank discounts the future.

We view this study to be part of a growing body of literature on the demand aspect of secular stagnation. Lastly, future work that investigates the elaborate mechanisms of demand- and supply-side interactions would be especially important. For example, the effect of an investment hangover on the supply side should be taken into account.

Appendix A

The explicit representations of the coefficient matrices and the vector in (11) and (12) are

\[
x_{t+1} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & : \\ 0 & 1 & 0 & \cdots \\ 0 & \cdots & : & 0 \\ 0 & \cdots & 0 & \rho_{\omega_{jt+1}} \end{bmatrix} x_t + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ : & 0 & : \\ 0 & 0 & 0 \end{bmatrix} z_t + i_t + \begin{bmatrix} 0 \\ 0 \\ : \\ 0 \\ 0 \end{bmatrix} \varepsilon_{t+1},
\]

\[
E_t \begin{bmatrix} \omega_{fjt} & 0 & 0 \\ \beta_{rjt} & \beta_{fjt} & 0 \\ 0 & 0 & \delta \end{bmatrix} z_{t+1} = \begin{bmatrix} -\hat{\omega}_{fjt} & 0 & 0 & 0 & -c_{\pi_{jt}} & 0 & 0 \\ 0 & -\hat{\beta}_{fjt} & \hat{\beta}_{gjt} & -\hat{\beta}_{rjt} & \hat{\beta}_{yjt} & 0 & 0 & -c_{g_{jt}} & -\phi_{jt} \\ 0 & \cdots & 0 & 0 & 0 & -1 \end{bmatrix} x_t + \begin{bmatrix} 1 & -\gamma_{jt} & -\xi_{jt} \\ 0 & 1 & -\theta_{jt} \\ 0 & 0 & 1 \end{bmatrix} z_t + i_t + \begin{bmatrix} 0 \\ \beta_{rjt} \\ 0 \end{bmatrix},
\]
where $\tilde{\omega}_{ft} = 1 - \omega_{ft}$, $\tilde{\beta}_{ft} = 1 - \beta_{ft}$, and $\tilde{\beta}_{yt} = 1 - \beta_{yt}$.

**Appendix B**

The loss function (18) of inflation targeting, including the penalty on interest rate variations, as in Williams (2012), is described as

\[
L^{IT}(\cdot) = \pi_t^2 + \lambda y_t^2 + \nu (i_t - i_{t-1})^2 + \psi_j i_t^2
\]

\[
= \begin{bmatrix}
\pi_t & y_t & i_t - i_{t-1} \\
y_t & \lambda & 0 \\
i_t & 0 & \nu & 0
\end{bmatrix} \begin{bmatrix}
\pi_t \\
y_t \\
i_t - i_{t-1}
\end{bmatrix}
\]

\[
= \begin{bmatrix}
x_t' \\
z_t \\
i_t
\end{bmatrix} \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \lambda & 0 & 0 \\
0 & 0 & \nu & 0
\end{bmatrix} \begin{bmatrix}
x_t' \\
z_t \\
i_t
\end{bmatrix}
\]

\[
= \begin{bmatrix}
O_{3 \times 3} & | & O_{3 \times 8} \\
| & \nu & 0 & 0 & 0 & 0 & 0 & 0 & -\nu \\
| & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
= \begin{bmatrix}
x_t' \\
z_t \\
i_t
\end{bmatrix} \begin{bmatrix}
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \lambda & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
| & -\nu & 0 & 0 & 0 & 0 & 0 & 0 & \nu + \psi_{jt}
\end{bmatrix}
\]
Similarly, that of the speed limit policy with the penalty on interest rate variations is described as

\[
L^{SLP}(\cdot) = \pi_t^2 + \lambda(y_t - y_{t-1})^2 + \nu(i_t - i_{t-1})^2 + \psi_j i_t^2
\]

\[
= \begin{bmatrix}
\pi_t \\
y_t - y_{t-1} \\
i_t - i_{t-1} \\
i_t
\end{bmatrix}^T \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \lambda & 0 & 0 \\
0 & 0 & \nu & 0 \\
0 & 0 & 0 & \psi_j
\end{bmatrix} \begin{bmatrix}
\pi_t \\
y_t - y_{t-1} \\
i_t - i_{t-1} \\
i_t
\end{bmatrix}
\]

\[
= \begin{bmatrix}
x_t' \\
z_t \\
i_t
\end{bmatrix}^{'} \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \lambda & 0 & 0 \\
0 & 0 & \nu & 0 \\
0 & 0 & 0 & \psi_j
\end{bmatrix} \begin{bmatrix}
x_t' \\
z_t \\
i_t
\end{bmatrix}
\]

\[
= \begin{bmatrix}
x_t \\
z_t \\
i_t
\end{bmatrix}^{'} \begin{bmatrix}
0 & \lambda & 0 & 0 & 0 & 0 & 0 & -\lambda & 0 & 0 \\
0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\nu & 0 & \nu & 0 & 0 & 0 & 0 & 0 & 0 & -\nu
\end{bmatrix} \begin{bmatrix}
x_t \\
z_t \\
i_t
\end{bmatrix}
\]

\[
= \begin{bmatrix}
x_t \\
z_t \\
i_t
\end{bmatrix}^{'} \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\lambda & 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\nu & 0 & 0 & 0 & 0 & 0 & 0 & \nu + \psi_j
\end{bmatrix} \begin{bmatrix}
x_t \\
z_t \\
i_t
\end{bmatrix}
\]

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Appendix C

In this appendix, we briefly explain the recursive saddle point method, which enables us to regard the original problem as a certain kind of recursive problem.

To do so, we first rewrite (16) as

\[
E_t H_{t+1} z_{t+1} = \mu_t,
\]

\[
0_{3 \times 1} = A_{21j_t} x_t + A_{22j_t} z_t + i_t b_{2j_t} - \mu_t,
\]

where \( \mu_t \) is an \( 3 \times 1 \) vector. Hence, \( z_t \) becomes a linear function of \( x_t, \mu_t, i_t, j_t \); that is

\[
z_t = \tilde{z}(x_t, \mu_t, i_t, j_t) \equiv A_{22j_t}^{-1} (-A_{21j_t} x_t + \mu_t - i_t b_{2j_t}).
\]

Moreover, we redefine the objective function as

\[
\tilde{L}(\tilde{x}_t, \mu_t, i_t, \gamma_t, j_t) \equiv L(x_t, \tilde{z}(x_t, \mu_t, i_t, j_t)) - \gamma_t \mu_t + \xi_{t-1} \frac{1}{\beta} H_{j_t} \tilde{z}(x_t, \mu_t, i_t, j_t)
\]

\[
\equiv \begin{bmatrix} \tilde{x}_t \\ i_t \end{bmatrix}' \hat{W}_{j_t} \begin{bmatrix} \tilde{x}_t \\ i_t \end{bmatrix},
\]

where \( \tilde{x}_t = [x_t', \xi_{t-1}]' \) and \( i_t = [\mu_t', i_t, \gamma_t']' \) are vectors of appropriate dimensions, \( \hat{W}_{j_t} \) is a matrix of appropriate dimension, and \( \xi_{t-1} \) and \( \gamma_t \) are additional state and control variables, respectively, satisfying \( \xi_t = \gamma_t \).

Then the dual-optimization problem, or the recursive saddle point problem, is given by

\[
\max_{\{\mu_t, i_t\}_{t=0}^\infty \{\gamma_t\}_{t=0}^\infty} \min_{E_0} \sum_{t=0}^\infty \beta^t \begin{bmatrix} \tilde{x}_t \\ i_t \end{bmatrix}' \hat{W}_{j_t} \begin{bmatrix} \tilde{x}_t \\ i_t \end{bmatrix},
\]

s.t. \( x_{t+1} = A_{11j_t+1} x_t + A_{12} \tilde{z}(x_t, \mu_t, i_t, j_t) + i_t b_1 + C_{j_t+1} \varepsilon_{t+1}, \)

\( \xi_t = \gamma_t. \)

Because our simulation assumes an unobservable regime, the probability distribution of the regime is considered as a state variable.\(^8\) Thus, the state vector is finally stacked as \( [\tilde{x}_t', \mu_t', \gamma_t']' \). The dual-optimization problem can be solved using algorithms to determine the solution, value functions, and optimal policy functions. The solution to the main problem can be obtained from that of the dual-optimization problem. See Svensson and Williams (2007) for further detail.

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\(^8\)Williams (2012) considers “observable regimes” as well as “unobservable regimes.” Here, we omit the observable cases, because there is a small uncertainty effect in that case.
References


Figure 1: Real GDP, real potential GDP, and output gap.

Notes: Real GDP and real potential GDP are taken from the website of the St. Louis Fed FRED. Output gap is percentage change of real GDP to real potential GDP.
Figure 2: Impulse response to an interest rate spreads shock. (A) Naive inflation targeting, \( \nu = \psi_1 = \psi_2 = 0 \). (B) Inflation targeting with interest rate smoothing, \( \nu = 0.5 \ (\psi_1 = \psi_2 = 0) \).
Figure 2: Continued. (C) Inflation targeting with penalty on interest rate volatility, \( \psi_1 = 0.1 \) and \( \psi_2 = 0.125 \) \((\nu = 0)\). (D) Inflation targeting with penalty on interest rate volatility, \( \psi_1 = 0.7 \) and \( \psi_2 = 0.875 \) \((\nu = 0)\).
Figure 3: Median impulse response of output gap to an interest rate spreads shock, calculated as deviations from constant model response. (A) Inflation targeting when $\psi_1 = \psi_2 = 0$. (B) Inflation targeting when $\nu = 0$ and $\psi_2$ is 25% larger than $\psi_1$. (C) Inflation targeting when $\nu = 0$ and $\psi_1 = \psi_2$. 
Figure 4: Impulse response to an interest rate spreads shock. (A) Naive speed limit policies, $\nu = \psi_1 = \psi_2 = 0$. (B) Speed limit policies with interest rate smoothing, $\nu = 0.5$ ($\psi_1 = \psi_2 = 0$).
Figure 4: Continued. (C) Speed limit policies with penalty on interest rate volatility, $\psi_1 = 0.1$ and $\psi_2 = 0.125$ ($\nu = 0$). (D) Speed limit policies with penalty on interest rate volatility, $\psi_1 = 0.7$ and $\psi_2 = 0.875$ ($\nu = 0$).
Figure 5: Median impulse response of output gap to an interest rate spreads shock, calculated as deviations from constant model response. (A) Speed limit policies when $\psi_1 = \psi_2 = 0$. (B) Speed limit policies when $\nu = 0$ and $\psi_2$ is 25% larger than $\psi_1$. (C) Speed limit policies when $\nu = 0$ and $\psi_1 = \psi_2$. 
Figure 6: Discount factor and impulse response of output gap to an interest rate spreads shock. (A) Inflation targeting. (B) Speed limit policies.