A Political Economic Analysis of Fiscal Gap

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ABSTRACT

We investigate tax policies of two governments hierarchically linked in a federation. At each level, policies can be influenced by lobbying activities of an interest group. We show that the sign of the fiscal gap depends on the influence of lobbying on government decisions and the institutional context (single-tier versus two-tier lobbying). In particular, lobbying at the state tier introduces a new ‘political’ vertical externality that contrasts the traditional fiscal externality. As a result the fiscal gap, and then the transfer from federal to state government, may have a positive sign in a second-best. This result is consistent with common observation but in contrast to previous theoretical analysis (Boadway and Keen, 1996) disregarding lobbying. Remarkably, lobbying taking place at both tiers reduces the relevance of the political externality and makes a negative fiscal gap more likely.

(JEL: D72, D78, H20, H71, H72, H77).
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1. Introduction

Normative analysis of vertical tax competition provides an intriguing result that contradicts standard tenets in the literature of fiscal federalism. In fact, it is generally believed that revenue should be prominently collected by the federal government for equity and efficiency reasons, while expenditure should be decentralized. According to this principle, the federal government collects more revenue than it is needed for federal expenditure - determining a positive fiscal gap - and then transfers a revenue share to the states. In an influential study, Boadway and Keen (1996) indicate otherwise. They show that concurrent taxation on a common base for federal and state governments determines a typical common pool problem leading to excessive state taxation. This kind of vertical externality can be corrected by a negative fiscal gap for the federal government (see also Keen, 1998). This surprising result, however, is generally unconfirmed by observed federal tax systems.

We extend Boadway and Keen (1996) introducing a political economy perspective, where decisions taken by state and federal governments can be influenced by an immobile interest group lobbying for tax reduction. To highlight the effects of two-tier lobbying, we first consider the case when just the state government is lobbied. In such a situation, we have two kinds of vertical externalities: the traditional fiscal externality and an additional ‘political’ externality caused by lobbying at a state level. The first vertical externality originates from the common tax base that induces the state government to an inefficiently high state tax and, then, forces the federal government to decrease the second-best federal tax (Boadway and Keen, 1996). The second vertical externality takes place because the interest group supports a reduction of the state tax. State lobbying, therefore, mitigates the effect of the first inefficiency. Depending on its

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1 An early contribution on vertically concurrent taxation is provided by Cassing and Hillman (1982).

2 Martinez (2008) includes a public input while Kotsogiannis and Martinez (2008) consider an ad valorem tax. Such generalizations may indeed lead to a positive fiscal gap. Keen and Kotsogiannis (2002) analyze the overall impact of fiscal externalities, since horizontal externalities make state taxes too low while vertical ones do the opposite. Keen and Kotsogiannis (2004) show that more intense fiscal competition, in form of a large number of states, is unambiguously harmful since it pushes taxes further away from the efficient equilibrium. Empirical support for vertical tax competition is provided by Besley and Rosen (1998) showing that an increase of the federal government tax rates on gasoline and cigarettes triggered an increase of state taxes in the US, between 1975 and 1989. For a similar result for income taxation in the US and Canada, see Esteller-Moré and Solé-Ollé (2001, 2002). Berry (2008) finds that tax rates are positively correlated to the number of tax authorities having overlapping jurisdiction in the same US county.
effectiveness, lobbying may even revert the incentive of the state to exploit the common tax base and ultimately force the federal government to a positive fiscal gap in a second-best. If, however, the interest group lobbies also the federal government to reduce taxation it is evident that a negative fiscal gap is more likely to emerge than in the case of lobbying restricted to the state level. This result shows that lobbying by ‘local’ interest groups, namely by groups that are not influential at the federal level, could improve the efficiency of the tax system.

Lobbying is envisaged to take place through the supply of campaign contributions, following the widely used model developed by Grossman and Helpman (1994) and based on the menu auction framework of Bernheim and Whinston (1986). Lai (2010, 2014) adopts the same model to investigate the impact of capital lobbying in a framework of horizontal asymmetric competition, showing results that contrast common wisdom on the impact of tax competition. We extend the Grossman and Helpman (1994) model to allow multi-tier lobbying as in Mazza and van Winden (2002, 2008). Compared to horizontal competition, the political economy literature on vertical tax competition is rather thin and based on the Leviathan model. Keen and Kotsogiannis (2003), incorporating both horizontal and vertical externalities, show that an increase in horizontal competition is welfare improving, in spite of the fact that raises revenues, in contrast with the Leviathan hypothesis. Wrede (2000) envisages federal and state governments sharing their tax base and providing productivity enhancing public services to increase their potential tax base. It is proved that, when only states supply public goods, over provision in a Nash equilibrium is possible, depending on the degree of complementarity between public goods and tax bases. This result would suggest to limit the overlapping of tax sources and expenditures. Also Esteller-Moré et al. (2012) investigate the problem of the assignment of public functions, focusing on tax powers, and - as in the present study – allow interest groups to lobby at different political levels for tax reductions. They show that interest group influence may be welfare improving and that taxation may be preferable at both federal and state levels rather than at a single tier.

The paper is organized as follows. Section 2 covers the basic model, while Section 3 includes a discussion of the results. Section 4 concludes the paper with some additional comments.

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3 The external effect from state level lobbying determines interesting results also in the case of full inter-jurisdictional labor mobility in contrast with previous literature (see section 3.3).

4 See also Grossman and Helpman (2002).
2. The model

We follow the model developed by Boadway and Keen (1996), in order to better highlight the extent of our generalization. We, therefore, consider a federation of $k$ states and $nk$ workers with identical preferences and immobile across states. Labor and an additional fixed factor, available in the same quantity in each state, are the inputs used for the production of a private good, $x$, and two public goods: a state public good, $g$, and a federal public good, $G$. The fixed factor is interpreted as foreign invested capital that does not move because of high sunk costs. The marginal rate of transformation between different public goods, each one of them and the private good is assumed equal to one. Public expenditure is financed by an income tax with a rate $\tau_L = t_L + T_L$ where $t_L$ is the state tax rate and $T_L$ is the federal tax rate. A worker’s preferences are described by the following separable utility function:

$$U = u(x, l) + b(g) + B(G)$$

(1)

where $l$ is labor supply, $u$ is a quasi-concave function, with $u_x > 0, u_l < 0$, where subscript refers to partial derivatives, and $b'(g) > 0, B'(G) > 0, b''(g) < 0$ and $G''(G) < 0$. Maximization of (1) over $x$ and $l$ subject to the budget constraint, $x = (w - \tau_L)l$, leads to the following first order condition:

$$\left(w - \tau_L\right)u_x + u_l = 0$$

(2)

which implies the labor supply function $l\left(w - \tau_L\right)$. We assume $l' > 0$, and substituting in (1), we obtain the following indirect utility function for a worker:

$$V = v\left(w - \tau_L\right) + b(g) + B(G)$$

(3)
where $u_l = v'$. Assuming an increasing and strictly concave production function $f(nl)$, which applies labor to the immobile factor, and a perfectly competitive labor market, the wage rate is:

$$w = f'(nl(w - \tau_L))$$

(4)

Consequently, the gross capital rent is:

$$r = f(nl(w - \tau_L)) - nl(w - \tau_L)f'(nl(w - \tau_L))$$

(5)

For future reference, we report the following comparative statics:

$$\frac{\partial w}{\partial \tau_L} \equiv w_{\tau_L} = \frac{-f'^{nl}l'}{1 - f'^{nl}l'} > 0 \text{ and } < 1, \quad \frac{\partial w}{\partial n} \equiv w_n = \frac{-w'l}{nl'} < 0$$

(6)

$$\frac{\partial r}{\partial \tau_L} \equiv r_{\tau_L} = \left(1 - \frac{\partial w}{\partial \tau_L}\right)f''l'l'l = \frac{n^2lf''l'}{1 - f'^{nl}l'} < 0, \quad \frac{\partial r}{\partial n} \equiv r_n = -\frac{r'l'}{nl'} > 0$$

(7)

Up to this point, results reproduce those in Boadway and Keen (1996). Recalling (7), next section introduces capital owners’ lobbying to reduce labor taxes. For this purpose, contrary to Boadway and Keen (1996) where the rent of the immobile factor is fully appropriated by federal and state governments and shared in an exogenous fashion, we assume that invested capital belongs to owners not resident or taxed in the federation, having a budget constraint $x = r$ and indirect utility function, $\omega(r)$. This assumption is consistent with a taxation system based on the residence principle, and justified by the fact that any attempt to influence policymaking would be frustrated if the whole gain for capital from lobbying is appropriated by the government. As references for the analysis that will follow, we include further comparative statics. Given the resource constraints and a vertical transfer $S$ from the federal government, the budget constraint of a state government is:

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7 The assumption of foreign capital immobility is also consistent with the common wisdom that foreign direct investments are believed to be rather resilient, although this hypothesis needs to be qualified according to regions and periods (see IMF, 2011). Becker and Rauscher (2013) adopt the same assumption to analyze the horizontal tax competition in dynamic model.
Straightforward effects of taxes and transfers on the supply of the state public good are:

\[
g_{t_s} = (w_{t_s} - 1)nt_s l' + nl
g_{L_s} = (w_{L_s} - 1)nt_s l' = g_{t_s} - nl
g_s = 1 = S\]

(9)

where \(g_{t_s}\) is ambiguous because of the combined revenue and tax effects, \(g_{L_s}\) is negative and indicates a fiscal externality when state tax rate \((t_s)\) is positive, and \(g_s\) is positive. On the other hand, the federal government budget constraint is:

\[
G(t_L, T_L, S) = nkT_L l'[w(t_L, n) - \tau_L] - kS
\]

leading to the following effects of taxes and subsidy on the supply of the federal public good:

\[
G_{t_L} = (w_{t_L} - 1)kT_L l' + knl
G_{L_L} = (w_{L_L} - 1)kT_L l' = G_{t_L} - knl
G_s = -k
\]

(11)

where \(G_{t_L}\) is ambiguous and \(G_{L_L}\) is negative, again for the externality effect, when federal tax rate, \(T_L\), is positive. As a benchmark for future results, we derive the social optimum under the assumption that all the policies are selected by a unitary government institution. Since states do not decide on taxes and expenditure, transfers are not considered, in this case. Therefore, denoting with \(q\) the shadow price of federal public goods, the program of the social planner is to maximize the utility of a representative worker (3):

\[
\max_{\tau_L, G, g} \left\{ \nabla\left(w(\tau_L, n) - \tau_L\right) + b(g) + B(G) + q\left[G + kg - kn\tau_L l[w(\tau_L, n) - \tau_L] - kr(\tau_L, n)\right] \right\}
\]

subject to the budget constraint for a unitary government
\[
G + kg = kn\tau_Ll(w(\tau_L,n) - \tau_L) + kr(\tau_L,n)
\]  

(13)

First order conditions are:

\[
u_x = nkq\left[f'nl' + \frac{\tau_{ll'}}{l} - 1\right], \quad b' = -kq, \quad B' = -q
\]  

(14)

From (14), we obtain the following the second-best outcome:

\[
\frac{nkB'}{u_x} = \frac{nb'}{u_x} = \frac{1}{1 - \frac{\tau_{ll'}}{l} - f'nl'}
\]  

(15)

This is the usual Samuelson rule indicating that the Pareto-efficient provision of each tier’s public good occurs when the sum of the marginal rates of substitution of each state public good \(g\) for the private good \(x\) should be equal to the sum of the marginal rates of substitution of the federal public good \(G\) for \(x\), and both should be equal to marginal cost of public funds using distortionary taxation. Next section shows the results relative to policy-making under lobbying by the capitalist.

3. Results under lobbying

To introduce lobbying into the model, we adopt the very influential and widely applied model of interest group influence developed by Grossman and Helpman (2002) building on Bernheim and Whinston (1986). We assume that a capital owner can attempt to influence policies by submitting a ‘menu’ of policy contingent contributions to the state and federal governments, each of them compensating the policymakers for the political costs of passing a bill in favor of the lobby. Our analysis of monopsonistic lobbying is relevant in case of particularistic policies, which are

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\[8\] For comparison’s sake, recall that the social optimum condition in in Boadway and Keen (1996) is:

\[
\frac{nkB'}{u_x} = \frac{nb'}{u_x} = \frac{1}{1 - \frac{\tau_{ll'}}{l}}
\]

from the concavity of \(B\) and \(b\) we see that public good provision decreases in our setting because we exclude rent tax as a source revenue and allow for (distortionary) labor taxes only. This increases the marginal cost of public funds.
exclusively offered to a lobby and impose widespread marginal costs over population that do not elicit counteracting opposition (see Baron, 1994; Grossman and Helpman, 1996).\(^9\)

At both government tiers, the capital owner profits from advocating a lower tax on labor, as it is evident from the negative impact on rent in (7). Thus, the latter offers (differentiable) tax contingent contributions, \(\zeta(t_L)\), to the state government and \(\theta(T_L)\) to the federal government \((\zeta_{t_L}, \theta_{t_L} < 0)\).\(^{10}\)

To investigate the effect of two-tier lobbying, an interesting case is when governments can tax just labor. Otherwise, the federal government could extract the full capital income, and then the (non-resident) capital owner would have no incentive to lobby because any potential increase of the capital rent (from lobbying) would be eventually taken away.

The sequence of events is as follows. At the first stage of policymaking, the federal government levies a tax on labor \((T_L)\). If the federal government is non benevolent and the capitalist has access at that political tier,\(^{11}\) his or her policy choice is preceded by the capitalist’s offer of a contribution schedule \(\theta^o(T_L)\), mapping every feasible federal tax on labor into a contribution. The contribution schedule is selected optimally; namely it maximizes the (net) utility of the capitalist. The federal government, then, chooses a tax, \(T_L^o\), maximizing the federal government policymaker’s objective function and obtains the corresponding (monetary equivalent) reward, \(\theta^o(T_L^o) \geq 0\), from the capitalist. At the second stage of policymaking, the state government levies a tax on labor \(t_L\) that maximizes that government’s objective function. Again, if the government is non benevolent, his or her decision is preceded by the capitalist’s offer of an optimal (for the capitalist) contribution schedule, \(\zeta^o(t_L)\), mapping every feasible state tax on labor into a contribution to the state government. Thus, the choice of a tax rate, \(t_L^o\), guarantees to the state policymaker a contribution equal to \(\zeta^o(t_L^o)\). To reduce notation, we omit the superscript \(^o\) from now on.

The assumption that interest groups may try to influence policymakers sequentially is justified by the fact that the public budgetary process is typically separated into successive stages. Moreover,

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\(^9\) We also investigate the case of workers’ lobbying. This generalization does not provide qualitatively different results. Calculations are available upon request.

\(^{10}\) Contributions can be generally interpreted as something beneficial for the receiver and costly for the donor. For example, favorable policies can be (implicitly) exchanged for campaign contributions, future employment opportunities (‘revolving doors’), elite services (e.g. parties, perks, holidays, tickets etc.), or even for bribes.

\(^{11}\) Throughout the paper, we assume that when the government is benevolent, no access is provided to influencing activities. On the contrary, if the government is non benevolent and thus willing to accept a contribution, there is no restriction to influence. Intermediate cases would be when lobbying is performed inefficiently (see, for example: Hillman and Riley, 1989; Mazza and van Winden, 2002, 2008).
since the capitalist may able to lobby just one policymaker, we distinguish the case of two-tier lobbying from the cases of influence restricted at a single stage. Sections 3.1 and 3.2. assume that labor is immobile, in order to concentrate on the effect of lobbying.

### 3.1. Policies with a benevolent federal government and a non-benevolent state government

**State government**

Starting from the lower tier of policymaking, taking the federal policy variables $T_L$ and $S$ as given, and assuming that the capital owner has political access to influence the (non-benevolent) state government, the latter will

$$
\max_{T_L} \nabla \psi(L(\tau_L) - \tau_L) + b(g) + B + \lambda \zeta(t_L)
$$

Subject to (8) and capitalist’s maximization condition:

$$
\omega' r_{t_L} - \zeta'(t_L) = 0
$$

From (16) and (17), we see that lobbying induces the policymaker to take into account the preferences of the (non-resident) capital. The weight attached to capital’s welfare, $\lambda > 0$, reflects the relative interest of the policymaker for the contribution with respect to residents’ social welfare. Although the capital owner lobbies to reduce the tax on labor, the policy is still suboptimal for the unorganized group and biased at the advantage of capital. Moreover, since the policymaker is just compensated for the biased policy, monopsonistic lobbying leads to a ‘full capture’ of the state policymaker by capital. After lobbying, the local government chooses a tax on labor that satisfies

$$
v'(w_{t_L} - 1) + b'g_{t_L}^{NB} + \lambda \omega' r_{t_L} = 0
$$

where superscript of $g_{t_L}^{NB}$ refers to the types of state government that chooses a policy, in this case a non-benevolent state government. Notice that, without lobbying, we would have $\lambda = 0$ and obtain the same result as presented in Boadway and Keen (1996). From (17) and the assumption on the utility function, it is straightforward to ascertain that lobbying has a negative impact on $t_L$, as expected. After rearranging, we obtain (see the Appendix):
\[
\frac{nb'}{u_x} = \frac{1}{1 - \tau_l l' f^* h l' + \frac{G_t}{(w_r - 1)knl}} \left( 1 - \frac{\lambda \omega f'}{u_x} f^* h^2 l' \right)
\]  

(19)

where the right hand side indicates the marginal cost of public funds (MCPF). Comparing (19) with the second best optimal decision (15) shows a vertical externality from state taxation on federal revenues, given by \(G_t\), which has a negative sign when \(T_l > 0\) and then pushes \(t_L\) above the second-best.\(^{12}\) The main novel element with respect to Boadway and Keen (1996) is the political externality from lobbying, which is equal to \(-\frac{\lambda \omega f'}{u_x} f^* h^2 l' > 0\) and always increases the MCPF inducing a lower \(t_L\). This result shows that lobbying tends to counterweigh the vertical externality of state taxes on federal revenue. From (19) and state government budget constraint (8), we get the optimal state tax rate selected by a non-benevolent state government, \(t_{L}^{\text{NB}}(T, S)\) (where superscript indicates non-benevolent government), which is supported in equilibrium by the contribution schedule:

\[
\zeta^* = \frac{1}{\lambda} \left\{ v(w(\tau_L; t_{L}^{B}) - \tau_L (t_{L}^{B})) + b(g; t_{L}^{B}) \right\} - \left\{ v(w(\tau_L; t_{L}^{NB}) - \tau_L (t_{L}^{NB})) + b(g; t_{L}^{NB}) \right\}
\]

(20)

The state tax rate \(t_{L}^{B}\) maximizes \[v(w(\tau_L) - \tau_L) + b(g)],\] i.e. the state government’s objective function without lobbying contributions. Comparison with (18) and the assumptions on utility functions indicate that \(t_{L}^{B} \neq t_{L}^{NB}\), implying \(\zeta^* > 0\). From (18), assuming that the second order condition for a maximum holds, we can also ascertain that:\(^{13}\)

\[
\frac{dt_{L}^{\text{NB}}}{d\lambda} = \frac{-\omega r_{t_i}}{n[w_r - 1 + v'(w_r - 1) + b'g_{t_i} + b'(g_{t_i})^2] + \lambda \left[ \omega r_{t_i}^2 + \omega r_{t_i}^{t_i} \right]} < 0
\]

(21)

\(^{12}\) Recall that \(\frac{G_t}{(w_r - 1)knl} = \frac{T_l l'}{l} > 0\).

\(^{13}\) From differentiating (18): 
\[
\left[ v'(w_r - 1) + b'g_{t_i} + b'(g_{t_i})^2 + \lambda \left[ \omega r_{t_i}^2 + \omega r_{t_i}^{t_i} \right] \right] dt = -\omega r_{t_i} d\lambda.
\]
The sign of (21) shows that, the impact of the relative preference of the state policymaker for lobbying contributions has a negative effect on labor tax, as we would expect.

Federal government
Recalling that the capital owner is non-resident, a benevolent federal government will then have the following program:

\[
\max_{T, S} \left[ \psi(w(T) - T) + b\left( t^{NB}_L, T, S \right) + B\left( t^{NB}_L, T, S \right) \right] \\
\text{s.t. } t_L = t^{NB}_L (T, S) 
\]  

(22)

The first order conditions for \( T_L \) and \( S \) are, respectively:

\[
\left( -\lambda \omega' r_L + B' g_L \right) \left( 1 + \frac{\partial t^{NB}_L}{\partial T_L} \right) + nl (kB' - b') = 0 \\
\left( -\lambda \omega' r_L + B' g_L \right) \left( \frac{\partial t^{NB}_L}{\partial S} \right) + (b' - kB') = 0 
\]  

(23)

From (23), we have

\[
\left( -\lambda \omega' r_L + B' g_L \right) \left( 1 + \frac{\partial t^{NB}_L}{\partial T_L} + nl \frac{\partial t^{NB}_L}{\partial S} \right) = 0 
\]  

(24)

Where

\[
1 + \frac{\partial t^{NB}_L}{\partial T_L} + nl \frac{\partial t^{NB}_L}{\partial S} = \frac{\left( w_r - 1 \right) b' n'l' + b'' g_n l}{v' w_{r, t, z} + v'(w_r - 1)^2 + b' g_{t, z} + b'' g_{z, t} \frac{L}{2} + \lambda \left( \omega'' (r_{t, z})^2 + \omega' r_{t, z} \right)} \\
- nl \frac{b'' g_{t, z}}{v' w_{r, t, z} + v'(w_r - 1)^2 + b' g_{t, z} + b'' g_{z, t} \frac{L}{2} + \lambda \left( \omega'' (r_{t, z})^2 + \omega' r_{t, z} \right)} \\
= \frac{\left( w_r - 1 \right) b' n'l'}{v' w_{r, t, z} + v'(w_r - 1)^2 + b' g_{t, z} + b'' g_{z, t} \frac{L}{2} + \lambda \left( \omega'' (r_{t, z})^2 + \omega' r_{t, z} \right)} > 0 
\]
and the denominator is negative by the second order condition in the state problem. Thus, the federal government policy has to satisfy the following condition:

$$G_{t}^{B,NB} = \frac{\lambda \omega' \tau_{L}^{T}}{B'(G^B)}$$ (25)

the first superscript refers to the type of federal government that chooses a policy, in this case a benevolent federal government, while the second one refers to state government, in this case a non-benevolent state government. Equation (25) highlights the external effect of lobbying at the state level on the federal government decision, showing a negative impact of state tax on the federal public good. If the state government were not lobbied, $G_{t}^{B,NB} = 0$ (as in Boadway and Keen, 1996) because the federal government would incorporate the fiscal externality when choosing $G$. However, since lobbying at the state tier (but not at the federal tier) has a negative impact on the state tax, the benevolent federal government tries to offset that effect by increasing the federal public good provision. Using (11), we obtain the optimal labor tax of the benevolent federal government:

$$T_{L}^{B,NB} = - \frac{\lambda \omega' \lambda_{f}}{kB'} > 0$$ (26)

As long as the state government is non-benevolent ($\lambda > 0$), this result contrast with the normative analysis of Boadway and Keen (1996) where federal tax is equal to zero. The federal government, which is not politically influenced by capital, reacts to the reduction of the state labor tax (due to state lobbying) by levying a tax on labor.

Substituting (26) into the federal budget constraint (10), we get an intergovernmental transfer to the state government:

$$S_{t}^{B,NB} = - \frac{\{ G_{t}^{B,NB} B' + \lambda \omega' \lambda_{f} \} f n^{2}}{kB'}$$ (27)

A central and somewhat counterintuitive result in Boadway and Keen (1996) is a negative $S$, namely a negative fiscal gap. In our study, the sign of fiscal gap depends on the conflict between
vertical fiscal externality and political externality due to the lobbying activity represented by \( \lambda \geq 0 \). If state government is not influenced by capital (\( \lambda = 0 \)) we have the well-known case of vertical fiscal externality with greater than optimal state tax rate; thus, the federal government will set an optimal consolidated tax rate by decreasing federal tax rate. In such a case, fiscal revenue of the federal government is insufficient and it has to receive a transfer from the state government: a negative fiscal gap then stems from vertical fiscal externality.

However, the outcome in our study differs when \( \lambda > 0 \). At the state tier, lobbying tends to reduce the tax on labor from (21). If, therefore, the impact of lobbying on reducing the state tax fully (or partially) offsets the incentive of the state government to overexploit the common tax base (namely to set an inefficiently high tax rate), then lobbying fully (or partially) internalizes the negative fiscal externality of state policy-making, and the fiscal gap is nil (or still negative) from (27).

If, on the other hand, lobbying has a sufficiently high impact on the state government policy, it could reduce the state labor tax so much to necessitate a positive fiscal gap to restore efficiency: the federal government increases the labor tax rate to subsidize the state government through an intergovernmental transfer. In the latter case, we can say that the strong preference of the state government for lobbying (\( \lambda \)) causes a net ‘political externality’ for the federal government, in the sense that the externality due to lobbying exceeds that caused by overexploitation of the common tax base. Thus:

\[
S_{B, NB}^{B, NB} = 0 \Leftrightarrow \lambda = -\frac{B'G_{B, NB}^{B, NB}}{\partial f_n^{*, 2f_n^{*, 2}}} > 0
\]

From the previous discussion, it is evident that a “positive fiscal gap” here stems from the interest that the state government has for campaign contributions, which are not accruing to the federal government. Moreover, for all values of \( \lambda \), the federal government can internalize the distortion of state labor tax by adjusting federal labor tax. Thus, federal government controls intergovernmental fiscal imbalance just through the intergovernmental transfer. Since this transfer is lump-sum fashion, even if the state government has interest for campaign contribution, the second best outcome is achieved, under lobbying at the state level (see the Appendix). Results under this specific regime of lobbying are summarized in the following Proposition.
PROPOSITION 1. If the capital owner lobbies the state government but not the federal government, a second best outcome is achieved and the sign of the federal transfer (i.e. fiscal gap) is ambiguous.

From Proposition 1 we can derive the following Corollary.

Corollary 1. If and only if the political externality due to state government lobbying cancels out the vertical fiscal externality, the federal transfer becomes zero and second best allocation is achieved by the state policy.

We will see in the next paragraph that when lobbying intervenes also at the federal tier it lessens the political externality effect.

3.2. Policies with non-benevolent federal and state governments.

We now investigate whether intergovernmental transfer always achieves a second best outcome. Thus, we consider the case where capitalist contributes both level of government, that is, the case of two-tier lobbying.\footnote{14}

Federal government

Once the state government will select a tax on labor satisfying (18) or (19), the federal government (or policymaker) problem is:\footnote{15}

\[
\max_{\tau_L, s} \left( w(\tau_L) - \tau_L \right) + B \left( t_L^{NB}(T_L, S) \right) + \phi \theta \left( T_L \right)
\]

\[
\text{s.t. } t_L^{NB} = t_L^{NB}(T_L, S) \tag{28}
\]

This time the capitalist lobbies both governments (policymakers). Therefore, the net utility of the capitalist is, recalling (20):

\footnote{14} It can be shown that when the federal government is only lobbied the sign of the fiscal gap is still ambiguous but, in this case, a second best allocation is not achieved. The intuition is that the state government, as a Stackelberg follower, cannot correct the distortion caused by lobbying at the federal level. Proof is available upon request.

\footnote{15} This problem differs from (22) because capital owner can lobby for both level of governments (two-tier lobbying), as in Mazza and van Winden (2002, 2008).
where $\zeta^*$ is as in (20) and $\theta(T_{L})$ is the contribution schedule offered to the federal policymaker. Optimization by the capitalist leads to:

$$\theta_{T_{L}} = \left( \frac{1}{\lambda} \right) \left[ b'(g^{N}) nl^{B} - b'(g^{NB}) n l^{NB} \right]$$

(30)

Lobbying at the federal level implies that $\theta_{T_{L}}$ is negative. Thus, the federal government (policymaker)'s first order conditions for $T_{L}$ and $S$ are respectively:

$$\left[ -\lambda \omega^{'} r_{T_{L}}^{NB} + B'(G^{NB}) G_{T_{L}}^{NB,NB} \left( 1 + \frac{\partial T_{L}^{NB}}{\partial T_{L}} \right) + nl^{NB} (kB'(G^{NB}) - b'(g^{NB})) \right] + \frac{\phi n}{\lambda} \left( b'(g^{B}) y^{B} - b'(g^{NB}) y^{NB} \right) = 0$$

and

$$\left[ -\lambda \omega^{'} r_{T_{L}}^{NB} + B'(G^{NB}) G_{T_{L}}^{NB,NB} \left( \frac{\partial T_{L}^{NB}}{\partial S} \right) + \left( b'(g^{NB}) - kB'(G^{NB}) \right) = 0 \right.$$

(31)

Combining the above conditions, a federal policy has to satisfy the following condition:

$$G_{T_{L}}^{NB,NB} = \frac{\phi n (b'(g^{NB}) y^{NB} - b'(g^{B}) y^{B})}{\lambda B'(G^{NB}) \left( 1 + \frac{\partial T_{L}^{NB}}{\partial T_{L}} + nl \frac{\partial T_{L}^{NB}}{\partial S} \right)} + \frac{\lambda \omega^{'} r_{T_{L}}^{NB,NB}}{B'(G^{NB})}$$

(32)

Contrary to (25), the sign of the right hand side of (32) is ambiguous (the first term is positive – see (24) and (30) - whereas the second one is negative). If the interest of the federal government (policymaker) for contributions ($\phi$) is sufficiently high - and recalling that lobbying supports tax reduction – that government may not be willing to counteract fully the state tax reduction by increasing the level of federal public good. In fact, from (32) we derive the following optimal federal tax on labor, assuming lobbying at both stage and using (11):
Again, we can distinguish two effects in (33). The first term shows the effect of lobbying on the federal policymaker and it is negative. While direct lobbying to the federal policymaker induces a reduction of the federal tax rate, the second term indicates that the effect of lobbying on the state government (legislator), which means that the federal tax tends to increase as a reaction to the reduction of the state tax due to lobbying at the state tier. Thus, the overall impact depends on the opposite effect of lobbying at both tiers. In conclusion, the federal tax rate induced by campaign contributions is higher or lower than the optimal level. Substituting (33) into the federal budget constraint (10), we get

\[
S_{NB,NB}^{NB} = \frac{1}{kB'(G^{NB})} \left\{ \frac{nl\phi(b'(g^{NB}) - b'(g^{B})}{k'} - \frac{\lambda k'(G^{NB})}{(B_k - 1)^{NB}} \left( 1 + \frac{\partial t_l^{NB}}{\partial T} + nl \frac{\partial t_l^{NB}}{\partial S} \right) \right\} (34)
\]

Similarly to the previous case of benevolent federal government, the sign of fiscal gap is again ambiguous. The intuition is that the stronger is the interest of the federal government for contributions ($\phi > 0$), the more likely is a negative fiscal gap, because the federal government is now more reluctant to collect labor tax revenue. From (31) it is easy to ascertain that the second best outcome cannot be achieved, since $b' \neq kB'$.

**PROPOSITION 2.** If the capital owner lobbies both state and federal government, a second best outcome cannot be achieved and the sign of the federal transfer (i.e. fiscal gap) is ambiguous.

**3.3. Policies with costless labor mobility.**

So far, we have assumed immobile labor. We now relax this restriction and allow workers to relocate costless among two states, $\alpha$ and $\beta$. In this case, states take residential mobility into account when they decide about their policies. To show the effect of labor mobility, we investigate the case when the capital owner has political access to influence just the non-benevolent state government.
Benevolent federal government and a non-benevolent state government

Again we follow the analytical framework adopted by Boadway and Keen (1996), in order to show how their results are affected by lobbying. We assume a total population $\bar{n}$ such that population of state $\beta$ equals $\bar{n} - n^\alpha$ where $n^i$ represents a total population of state $i$. Thus, migration equilibrium implies

$$\nu[w(\tau^\alpha_L, n^\alpha) - \tau^\alpha_L] + b\left(g^\alpha\right) = \nu[w(\tau^\beta_L, \bar{n} - n^\alpha) - \tau^\beta_L] + b\left(g^\beta\right)$$

(35)

where $\tau^i_L$ and $g^i$ represent consolidated tax rate and state public goods in state $i$ ($i = \alpha, \beta$).

From this equation, the migration function becomes $n^\alpha = n^\alpha(\tau^\alpha_L, g^\alpha; \tau^\beta_L, g^\beta, \bar{n})$, where $g^\beta$, $\tau^\beta_L$ and $\bar{n}$ are exogenous variables. To see the effect of labor tax on migration, from (35):

$$\left\{v^\alpha_w w^\alpha_w + b^\alpha g^\alpha_w - (v^\beta_w w^\beta_w n^\beta - b^\beta_w g^\beta_w n^\beta)\right\}dn^\alpha =$$

$$-\left\{v^\alpha_w (w^\alpha_w - 1) + b^\alpha g^\alpha_w \right\}dt^\alpha + \left\{v^\beta_w (w^\beta_w - 1) + b^\beta_w g^\beta_w \right\}dt^\beta$$

$$- b^\alpha_dS^\alpha + b^\beta_dS^\beta$$

(36)

Define $\Delta \equiv v^\alpha_w w^\alpha_w + b^\alpha g^\alpha_w - (v^\beta_w w^\beta_w + b^\beta_w g^\beta_w) n^\beta = v^\alpha_w w^\alpha_w + b^\alpha g^\alpha_w + v^\beta_w w^\beta_w + b^\beta_w g^\beta_w = 2(v^\alpha_w w^\alpha_w + b^\alpha g^\alpha_w)$. Notice that $\bar{n} - n^\alpha = n^\beta$ and $\frac{\partial n^\beta}{\partial n^\alpha} = -1$. Stability condition requires that the welfare impact of immigrants on residents is negative, namely $\Delta < 0$. Thus, the effects of labor taxes on migration are:

$$\frac{dn^i}{dt^i_L} = -\frac{1}{\Delta} \left\{v^i_w (w^i_{\tau^i_L} - 1) + b^i g^i_{\tau^i_L}\right\}, \quad \frac{dn^i}{dT^i_L} = \frac{1}{\Delta} \left\{v^i_w (w^i_{\tau^i_L} - 1) + b^i g^i_{\tau^i_L}\right\}$$

$$\frac{dn^i}{dS^i} = \frac{1}{\Delta} \left\{v^i_w (w^i_{\tau^i_L} - 1) + b^i g^i_{\tau^i_L}\right\} - \left\{v^i_w (w^i_{\tau^i_L} - 1) + b^i g^i_{\tau^i_L}\right\} = 0$$

(37)

$$\frac{dn^i}{dS^j} = -\frac{b^i}{\Delta} > 0, \quad \frac{dn^i}{dS^j} = \frac{b^i}{\Delta} < 0$$

16
where \( i = \alpha, \beta \) (\( i \neq j \)), \( g'_{n_i} = t'_i l_i (1 - w'_i) \) and \( g'_{n_j} = n'_i l_i + n'_i l_j (w'_i - 1) \). To ascertain the signs of \( \frac{dn'_i}{dt'_i} \) and \( \frac{dn'_j}{dt'_j} \) we first need to solve the maximization problems for the governments. Starting from the state government, the objective function and constraints of state are:

\[
\max_{t'_i} v\left(w'_i \left( t'_i, n'_i \right) - t'_i \right) + b\left(g'_i, n'_i, T_i \right) + B(G) + \xi_i'(t'_i) \\
\text{s.t.} \quad n'_i = n'_i \left(t'_i, t'_j, S'_i, S'_j, T_i \right), \quad g'_i = g\left(t'_i, T_i, S'_i, n'_i \right)
\]

where the contribution \( \xi_i'(t'_i) \) is derived from the maximization problem of the lobbying capitalist.

The first order condition is:

\[
v'_w \left(w'_i + w'_n n'_i - 1 \right) + b'_i \left(g'_i + g'_n n'_i \right) + \lambda_i' \omega'_i \left(r'_i + r'_n n'_i \right) = 0
\]

(39)

First two terms show the opportunity cost of reducing tax, while the last term is the marginal benefit accruing to the state policymaker in terms of contributions for tax reduction. Notice that the state policymaker chooses a suboptimal tax rate because of capital lobbying. Rearranging (39), we obtain:

\[
\frac{n'_b}{u'_s} = \frac{1}{1 - \frac{\lambda_i' \omega'_i}{u'_s} - f'_h l'_r + \frac{G_{t_i}}{w_{t_i} - 1} n'_{t_i}} \left[ 1 - \left( \frac{\lambda_i' \omega'_i}{u'_s} f'\left(n'_i \right) l'_i \right) - \frac{\Delta}{v'_w w'_n + b'_i g'_i - \lambda_i' \omega'_i r'_i} \right]
\]

(40)

Thus, comparing with (19), we can find that labor movement affects the magnitude of state tax and public goods. In particular, if \( v'_w w'_d + b'_d g'_d + \lambda_i' \omega'_i r'_i < 0 \) the impact of state policymakers’ preferences for political influence on labor mobility, \( \lambda_i' \omega'_i r'_i \), is higher (equal, smaller) than

\[
\text{when } v'_w w'_d + b'_d g'_d + \lambda_i' \omega'_i r'_i = 0, \text{ MRS corresponds to (19) because this condition means that the effect of labor mobility cannot affect the optimal condition of state government as in (39).}
\]

\[16\] See the Appendix.

\[17\] This is equivalent to \( 0 < \frac{\Delta}{v'_w w'_d + b'_d g'_d - \lambda_i' \omega'_i r'_i} > (\pm, <) \).

\[18\] When \( v'_w w'_d + b'_d g'_d + \lambda_i' \omega'_i r'_i = 0 \), MRS corresponds to (19) because this condition means that the effect of labor mobility cannot affect the optimal condition of state government as in (39).
the impact of labor mobility on the workers’ welfare – implying that the relative political influence of capital on the state government increases (stays unchanged, decreases) – then the state tax rate increases (stays unchanged, decreases).

Tax also induces labor to migrate \( \frac{dn}{dt_L} = -\frac{1}{\Delta} \left\{ v_w (w_{rt} - 1) + b' g_{it} \right\} = \left( \frac{\Delta_{\nu w} r_{it}'}{v_w' w_{rt} + b' g_{it}' - \Delta_{\nu w} r_{it}'} \right) > 0 \)

and influences the benefit of capital from lobbying\(^ {19} \). Firstly, as for the effect of migration, \( \frac{dn}{dt_L} \), the sign of \( v'(w_{rt} - 1) + b' g_{it}' \) has to be positive because numerator and denominator are negative from (7) and the stability condition for \( \Delta \). The intuition of the counterintuitive comparative statics, according to which a rise of the labor tax increases the amount of labor at the margin, is due to the fact that capital lobbying reduces tax to a suboptimal level for the workers. However, when free labor mobility is allowed, the state government can increase the tax rate, towards the optimal level for the workers, and attract new workers. In contrast, \( \frac{dn}{dt_L} < 0 \) from the result above.

Secondly, as for the effect on labor tax of an interest of campaign contribution, assuming that the second-order condition is satisfied, we have \( \frac{dn_{\text{LB}}}{d^2 \xi} = -\frac{\eta'}{\eta} (r_{it} + r_{it}' n_{it}'), \)

where

\( \eta = \left[ v_w' w'_{rt} + v_w' (w_{rt} - 1)^2 + v_w' w_{rt} n_{it}' + v_w' (w_{rt} - 1)^2 n_{it} + b' \left( g_{it}'(t_{it}) + g_{it}'(t_{it}) + b_n (g_{it}'(t_{it}) + (g_{it}')^2 n_{it}) \right) + \right. \)

\( \left. \frac{d^2}{dt_L^2} \right] < 0 \) and subscript “mobile” means the case of labor mobility. Recalling (39), \( r_{it} + r_{it}' \frac{dn}{dt_L} \) represents the total effect of a change of state tax on capital rent. The first term measures the negative direct effect on (17), while the second term reflects the positive indirect effect through labor migration. The latter positive indirect effect, obtained through the positive impact of tax on migration, increases capital’s marginal benefit of lobbying the state policymaker. Thus, the effect that relative preference of the state policymaker for lobbying has on labor tax depends on these two effects.

In conclusion, (40) does not imply an efficient allocation for the state government as in Boadway and Keen (1996) (see also Sato, 2000), where state government achieves efficient allocation

\(^{19} \) See the Appendix.

\(^{20} \) Differentiating (39): \( \frac{d}{dt_L} \left[ v_w' w_{rt} + v_w' (w_{rt} - 1)^2 + v_w' w_{rt} n_{it}' + v_w' (w_{rt} - 1)^2 n_{it} + b' \left( g_{it}'(t_{it}) + g_{it}'(t_{it}) + b_n (g_{it}'(t_{it}) + (g_{it}')^2 n_{it}) \right) + \right. \)

\( \left. \frac{d^2}{dt_L^2} \right] < 0 \) and subscript “mobile” means the case of labor mobility. Recalling (39), \( r_{it} + r_{it}' \frac{dn}{dt_L} \) represents the total effect of a change of state tax on capital rent. The first term measures the negative direct effect on (17), while the second term reflects the positive indirect effect through labor migration. The latter positive indirect effect, obtained through the positive impact of tax on migration, increases capital’s marginal benefit of lobbying the state policymaker. Thus, the effect that relative preference of the state policymaker for lobbying has on labor tax depends on these two effects.

In conclusion, (40) does not imply an efficient allocation for the state government as in Boadway and Keen (1996) (see also Sato, 2000), where state government achieves efficient allocation
through labor mobility, that is, “incentive equivalency”, and Nash equilibrium of competing regions is Pareto optimal (Myers, 1990) since $v^\prime(\{w^j_{t_k}-1\})+b^\prime g^j_{t_k}=0$. In our model, that happens if and only if $\lambda^j=0$ (no lobbying).\footnote{If $\lambda^j=0$, then $v^\prime(\{w^j_{t_k}-1\})+b^\prime g^j_{t_k}=0$ since $v^\prime j^\prime w^\prime j^\prime + b^\prime j^\prime g^\prime j^\prime \neq 0$ from the stability requirement $\Delta \equiv v^\prime\{w^\prime j^\prime w^\prime j^\prime + b^\prime j^\prime g^\prime j^\prime\}h^\prime j^\prime < 0$.} However, for $\lambda^j > 0$, the state government has an interest in campaign contributions and, therefore, have to attention the effect that mobility has on them through the change in the rent for the capital owner supplying those contributions. We then show that, under labor mobility, an efficient allocation cannot be derived, in contrast to previous literature. This counterintuitive result is summarized by the following Lemma.

**LEMMA 1.** When labor is perfectly mobile between states, an efficient allocation cannot be derived by the state government tax.

The question is now to ascertain whether a benevolent federal government can correct the tax distortion caused by lobbying at the state tier and achieve the second-best outcome. Taking the choices of the state government into account, the federal government will face the following program:

$$\max_{T,S,S^\prime} \delta[v^\alpha(w(T^\alpha, n^\alpha) - \tau^\alpha) + b(g(T^\alpha, T^\alpha, S^\alpha, n^\alpha) + B(G))]$$

$$+ (1 - \delta)[v^\beta(w(T^\beta, n^\beta) - \tau^\beta) + b(g(T^\beta, T^\beta, S^\beta, n^\beta) + B(G))]$$

$$s.t. \quad T^j(T, S^j, S^j), \quad n^j(T^j(T, S^j, S^j), T^j(T, S^j, S^j) T, S^j), \quad G = G[T, S^j, S^j]$$

We assume that both states are symmetric, namely: $v^j = v^j \equiv v$ and $b^j = b^j \equiv b$ have to hold in equilibrium. Thus federal government problem becomes as follows:

$$\max_{T,S,S^\prime} \delta[v^\alpha(w(T^\alpha, n^\alpha) - \tau^\alpha) + b(g(T^\alpha, T^\alpha, S^\alpha, n^\alpha) + B(G))]$$

$$s.t. \quad T^j(T, S^j, S^j), \quad n^j(T^j(T, S^j, S^j), T^j(T, S^j, S^j) T, S^j), \quad G = G[T, S^j, S^j]$$
From first order conditions, we get:

\[
\begin{align*}
\{v' \left( w' - 1 \right) + b' g' + B' G \left[ 1 + \frac{\partial \psi}{\partial t} + nl \frac{\partial \psi}{\partial s} \right] \} = 0 
\end{align*}
\]  

(42)

Since \(1 + \frac{\partial \psi}{\partial t} + nl \frac{\partial \psi}{\partial s} \neq 0\), the federal government has to satisfy:

\[
G_{iL, \text{mobile}}^{B, NB} = \left( \frac{v' w' + b' g' + b' w_l' + b' g_l^j}{v' w_n' + b' g_n^j - \lambda' r_l'} \right) \left( \frac{\lambda' r_l'}{B'} \right)
\]

(43)

where \(v' w' + b' g' = v' w_n' + b' g_n'\) from assumption. This leads to the following federal tax under labor mobility:

\[
\tau_{iL, \text{mobile}}^{B, NB} = -\frac{\lambda' r_l'}{2B'} \left( \frac{v' w_n' + b' g_n'}{v' w_n' + b' g_n' - \lambda' r_l'} \right) > 0
\]

(44)

Comparing (43) and (44) with (25) and (26), we see that labor mobility causes a higher (equal, lower) federal tax rate – and then federal public good - if \(\frac{\Delta}{v' w_n' + b' g_n'} > (\leq, \leq)\). Thus, the federal government has an opposite reaction to that of the state government.

The intuition is as follows. Capitalist lobbying for a state tax causes a sub-optimally low tax rate. If the wage tax rate further reduces (increases), labor will leave (enter) the state causing a loss (gain) for the capitalist, who also becomes less (more) powerful politically. Then the state government will set a higher (lower) tax rate rather than the case of immobile labor. Consequently, the benevolent federal government has now to reduce (increase) the federal tax (and public good) with respect to the case of immobile labor, in order to compensate for the change in the state tax rate. In this way, the federal government is able to correct the distortion from state tax rate and achieves the second best outcome. Substituting (44) into the federal budget constraint (10), we get

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22 See the Appendix.

23 See equation (40).
where $L = \frac{\Delta}{v_n^i w_n^i + b_n^j g_n^j - \lambda \omega^i r_n^j}$. Although the sign of (45) is ambiguous, when $\lambda^i$ satisfies the second best outcome (15), the federal government does not need to give the fiscal transfer, $S_{\text{mobility}}^{B,NB} = 0$. When $\lambda$ is zero (or approximating it), lobbying does not take place (or is of little relevance) and the transfer is negative, in line with Boadway and Keen (1996). Results are then summarized by the following Proposition.25

PROPOSITION 3. If capital lobbies the state government but not the federal government, and labor is mobile, the second best outcome is achieved and the sign of the federal transfer (fiscal gap) is ambiguous.

This result shows that, even in presence of labor mobility and lobbying, federal government can correct state distortion by a federal tax on labor wage and an intergovernmental transfer. The intuition is the same as in section 3.1.

4. Concluding comments

Our study investigates the effect of lobbying in the event of negative fiscal externalities due to a common tax base. The results show that lobbying at the state level introduces an additional vertical ‘political’ externality that can counteract the vertical ‘fiscal’ externality and sustain a positive fiscal gap, in line with what we observe in reality. The introduction of lobbying towards the federal

24 As in last section, if $\lambda^i$ can be set to satisfy the condition of

\[
\frac{1}{1 - \frac{r_n^j}{I_n^j} - f n^i t^i} = \frac{1}{1 - \frac{r_n^j}{I_n^j} - f n^i t^i + \left(\frac{\lambda \omega^i}{u^i} f^*(n^i)^2 I_n^j\right)^2 \left(v_n^i w_n^i + b_n^j g_n^j - \lambda \omega^i r_n^j\right)}
\]

can confirm $S_{\text{mobility}}^{B,NB} = 0$ (fiscal gap is zero).

25 See the Appendix.
government induce the latter to partially internalize the political externality making the positive fiscal gap more likely than in the previous case.

Throughout the analysis, we have assumed that the federal government is lobbied by capital when also the state government is subject to lobbying. Two further extensions have been explored. 26 Firstly, in the special case when capital is able to lobby the federal government but has no access to the state government, we can show that the second-best outcome is not achieved. The reason is that the federal government acts a Stackelberg leader and distortions from lobbying at the federal level cannot be internalized by the state government, through lobbying. Secondly, if we consider lobbying by unionized resident workers - in which case foreign capital does not lobby - qualitatively similar results are obtained in this paper. We leave to future research the case of a common rent tax (and wage tax) levied by the federal and state governments that are influenced by lobbying.

APPENDIX

Appendix A

Policymaking with a benevolent federal government and a non-benevolent state government

Equation (19)

To derive (19), we first substitute \( g_{rl} = (w_{rl} - 1)nt_{l}l' + nl \) and \( u_{l}l = v' \) into (18), obtaining

\[
(w_{rl} - 1)\left[u_{l}l + b'(nl + \frac{nl'}{w_{rl} - 1}) + \lambda\omega'r_{rl} = 0.\right.
\]

Then, after dividing both sides by \((w_{rl} - 1)\) and recalling, from (6), that \( \frac{f'}{w_{rl} - 1} = f'n' - 1 \) we get:

\[
\frac{u_{l}l + b'(nl + nl + f'n' - 1)}{\frac{\lambda\omega'r_{rl}}{w_{rl} - 1}} = 0\]

After rearranging and dividing by \( u_{l}l \left( \frac{f'n' - 1}{l'} + f'n' - 1 \right) \), we have:

26 Derivations of results are available upon request
\[
\frac{nb'}{u_x} = \frac{1}{1-\frac{\tau_i l'}{l}-f'nl'} + \frac{\lambda \omega r_e}{u_x \left( 1 - \frac{\tau_i l'}{l} - f'nl' \right) (w_{\tau_e} - 1)}
\]

\[
= \frac{1}{1-\frac{\tau_i l'}{l}-f'nl'} \left( 1 + \frac{\lambda \omega r_e}{u_x \left( w_{\tau_e} - 1 \right)^2} \right)
\]

From (7):

\[
\left( \frac{r_e}{l-w_{\tau_e}} \right) = f'n^2 l'
\]

(A3)

After substituting (A3) in (A2), we obtain

\[
\frac{nb'}{u_x} = \frac{1}{1-\frac{\tau_i l'}{l}-f'nl'} \left( 1 - \frac{\lambda \omega}{u_x f'n^2 l'} \right)
\]

(A4)

After adding and subtracting \(\frac{T_i l'}{l}\) to the denominator of (A4), and recalling from (11) that

\[
T_i l' = \frac{G_{\tau_e}}{(w_{\tau_e} - 1) k n}
\]

we substitute in (A4) obtaining (19).

**Second-best outcome**

From (25): \(G_{\tau_e}^{\text{B,NB}} = \frac{\lambda \omega r_e^{\text{B,NB}}}{B'}\). Substituting (25) into (23) we get \(b' = kB'\). Using this result and

(25) \(\lambda \omega = \frac{B'G_{\tau_e}^{\text{B,NB}}}{r_e^{\text{B,NB}}}\), we can rewrite (19) as

\[
\frac{nkB'}{u_x} = \frac{1}{1-\frac{\tau_i l'}{l}-f'nl'} \left( 1 - \frac{G_{\tau_e}}{u_x \left( r_e^{\text{B,NB}} / B' \right) f'n^2 l'} \right)
\]

(A5)
And substituting for $r_{x}$, we get

$$\frac{nkB'}{u_x} = \frac{1}{1 - \frac{\tau_{x}l'}{l} - f''nl' + \frac{G_{t_{x}}}{(w_{r_{x}} - 1)knl}} \left\{ 1 - \frac{B'G_{t_{x}}}{u_x(1 - w_{r_{x}})} \right\}$$

or, after rearranging:

$$\frac{nkB'}{u_x} = \frac{1}{1 - \frac{\tau_{x}l'}{l} - f''nl' + \frac{G_{t_{x}}}{(w_{r_{x}} - 1)knl}} \left[ 1 + \left( \frac{nkB'}{u_x} \right) \left\{ \frac{G_{t_{x}}}{(w_{r_{x}} - 1)knl} \right\} \right]$$

(A6)

And thus,

$$\frac{nkB'}{u_x} \left\{ 1 - \frac{G_{t_{x}}}{(w_{r_{x}} - 1)knl} \right\} = \frac{1}{1 - \frac{\tau_{x}l'}{l} - f''nl' + \frac{G_{t_{x}}}{(w_{r_{x}} - 1)knl}}$$

(A7)

From (A7):

$$\frac{nkB'}{u_x} \left\{ 1 - \frac{\tau_{x}l'}{l} - f''nl' \right\} = \frac{1}{1 - \frac{\tau_{x}l'}{l} - f''nl' + \frac{G_{t_{x}}}{(w_{r_{x}} - 1)knl}}$$

(A8)

Multiplying both sides of (A8) by $\left( 1 - \frac{\tau_{x}l'}{l} - f''nl' + \frac{G_{t_{x}}}{(w_{r_{x}} - 1)knl} \right) \left( 1 - \frac{\tau_{x}l'}{l} - f''nl' \right)$, we have the second best outcome (15):

$$\frac{nkB'}{u_x} = \frac{n'b'}{u_x} = \frac{1}{1 - \frac{\tau_{x}l'}{l} - f''nl'}$$
Corollary 1

We now prove that second best outcome is achieved also if \( S^{B, NB} = 0 \).

Firstly, we find the optimal \( \lambda^{SB} \) which achieves the second best outcome. Secondly, we show that \( \lambda^{SB} \) is just same \( \lambda^{B, NB=0} \) which satisfies \( S^{B, NB} = 0 \). Then, in order to find \( \lambda \) which achieves the second best outcome (15), we begin by setting \( \lambda \) that satisfies the following condition:

\[
\frac{1}{1 - \frac{\tau_l l'}{l} - f' h l'} = \frac{1}{1 - \frac{\tau_l l'}{l} - f' h l'} + \frac{G_{1}}{(w_{z} - 1)knl} \left( 1 - \frac{\lambda \omega'}{u_{c}} f' h^{2} l' \right)
\]

(A.9)

Equation (A.9) states that \( \lambda \) has to be such that the second best outcome (15), in the left hand side, has to equal the state government second best outcome (19), in right hand side.

Thus,

\[
\lambda^{SB} = \frac{G_{1}}{(w_{z} - 1)knl} \left( 1 - \frac{\tau_l l'}{l} - f' h l' \right) = - \frac{u_{c}}{\omega f' h^{2} l'} \frac{T_{l} l'}{l} = - \frac{u_{c} T_{l} l'}{\omega f' h^{2} l' \left( 1 - \frac{\tau_l l'}{l} - f' h l' \right)} > 0
\]

where \( \lambda^{SB} \) represents the optimal value when vertical externality can be fully internalized by political externality. Furthermore, we show that \( \lambda^{SB} \) is equal to \( \lambda_{S=0}^{B, NB} = - \frac{B_{0} G_{SB}^{B, NB}}{\omega f' h^{2} l^{2}} > 0 \) corresponding to the state government second best outcome when \( S^{B, NB} = 0 \) (27). In fact:

\[
\lambda^{SB} - \lambda_{S=0}^{B, NB} = - \frac{u_{c} T_{l} l'}{\omega f' h^{2} l' \left( 1 - \frac{\tau_l l'}{l} - f' h l' \right)} + \frac{B_{0} G_{SB}^{B, NB}}{\omega f' h^{2} l^{2}}
\]

Substituting \( nkT_{l} l \) into \( G_{SB}^{B, NB} \) from budget constraint of federal government when \( S^{B, NB} = 0 \):
\[ \lambda^{SB} - \lambda^{B, NB}_{S>0} = - \frac{u_i^* T_{iL}}{\partial f^* h^* l^2 \left( \frac{1 - \tau^*_L}{l - f^* h^* l} \right)} + \frac{B^T_{iL} n_{kL}}{\partial f^* h^* l^2} = - \frac{u_i^* T_{iL}}{l \partial f^* h^* l} \left( \frac{1}{1 - \frac{\tau^*_L}{l - f^* h^* l}} \right) - \frac{n_k B^*'}{u} = 0 \]

The last bracket in right hand side is zero because of the second best outcome (15), thus when \( \lambda = \lambda^{SB} \), \( S^{B, NB} = 0 \).

Policies with costless labor mobility

Equation (40)

Rearranging (39), \( v_{iL}^* \left( w_{iL}^* - 1 \right) + b_i^* g_i^* + \left( v_{iL}^* w_{iL}^* + b_i^* g_i^* + \lambda^* \omega^* r_i^* \right) n_{iL}^* = - \lambda^* \omega^* r_i^* \). Substituting

\[ n_{iL}^* = - \frac{1}{\lambda} \left\{ v_{iL}^* \left( w_{iL}^* - 1 \right) + b_i^* g_i^* \right\} , \]

we get \( v_{iL}^* \left( w_{iL}^* - 1 \right) + b_i^* g_i^* \left[ \frac{v_{iL}^* w_{iL}^* + b_i^* g_i^* - \lambda^* \omega^* r_i^*}{\Delta} \right] = - \lambda^* \omega^* r_i^* \). And thus \( v_{iL}^* \left( w_{iL}^* - 1 \right) + b_i^* g_i^* = \frac{- \lambda^* \omega^* r_i^*}{\Delta} \).

Moreover, we substitute \( g_i^* = n_i^* I_{iL}^* \left( w_{iL}^* - 1 \right) + n_i^* I_{iL}^* \) and \( n_i^* I_{iL}^* = v_{iL}^* \) into this equation, obtaining

\[ \left( w_{iL}^* - 1 \left[ u_i^* I_{iL}^* + b_i^* \left( n_i^* I_{iL}^* + \frac{n_i^* l_{iL}^*}{w_{iL}^* - 1} \right) \right] \right) = \frac{- \lambda^* \omega^* r_i^*}{\Delta} \left[ v_{iL}^* w_{iL}^* + b_i^* g_i^* - \lambda^* \omega^* r_i^* \right] . \]

Then, as in the case of no labor mobile, after dividing both sides by \( \left( w_{iL}^* - 1 \right) \) and substituting \( \frac{1}{w_{iL}^* - 1} = f_{iL}^* I_{iL}^* - 1 \), we get

\[ u_i^* I_{iL}^* + b_i^* \left( n_i^* I_{iL}^* + n_i^* l_{iL}^* (f_{iL}^* I_{iL}^* - 1) \right) = \left( \frac{- \lambda^* \omega^* r_i^* \Delta}{w_{iL}^* - 1} \right) \left[ \frac{v_{iL}^* w_{iL}^* + b_i^* g_i^* - \lambda^* \omega^* r_i^*}{\Delta} \right] . \]

We get:

\[ \frac{n_i^* b_i^*}{u_i^*} = \frac{1}{1 - \frac{\tau^*_L}{l - f_{iL}^* I_{iL}^*} + \frac{c_i^*}{w_{iL}^* - 1}} \left[ 1 - \left( \frac{\lambda^* \omega^*}{u_i^*} \right) \left( \frac{(n_i^*)^2 I_{iL}^* \Delta f^*}{v_{iL}^* w_{iL}^* + b_i^* g_i^* - \lambda^* \omega^* r_i^*} \right) \right] . \]

Derivation of sign of \( \frac{d n_i^*}{d t_{L}^*} \) in footnote 19

Introducing \( \frac{d n_i^*}{d t_{L}^*} = - \frac{1}{\Delta} \left( v_{iL}^* \left( w_{iL}^* - 1 \right) + b_i^* g_i^* \right) \) into first order condition (39), we can rewrite this as:
\[ v^i_w \left( w^i_{r_1} - 1 \right) + b^{i'} g^{i'}_i + \lambda \omega^j r^j_{r_1} - \left( v^i_w w^i_{r_1} + b^i g^i_{r_1} + \lambda \omega^j r^j_{r_1} \right) \frac{1}{\Delta} \left( v^i_w \left( w^i_{r_1} - 1 \right) + b^i g^i_{r_1} \right) = 0 \]

\[ \left( v^i_w \left( w^i_{r_1} - 1 \right) + b^i g^i_{r_1} \right) \left[ 1 - \left( v^i_w w^i_{r_1} + b^i g^i_{r_1} + \lambda \omega^j r^j_{r_1} \right) \frac{1}{\Delta} \right] + \lambda \omega^j r^j_{r_1} = 0 \]

Thus, \[ v^i_w \left( w^i_{r_1} - 1 \right) + b^i g^i_{r_1} = \frac{\lambda \omega^j r^j_{r_1} \Delta}{v^i_w w^i_{r_1} + b^i g^i_{r_1} + \lambda \omega^j r^j_{r_1}} \]. We then substitute this expression into (37),

\[ \frac{dn^i}{dt_L} = - \frac{1}{\Delta} \left( v^i_w \left( w^i_{r_1} - 1 \right) + b^i g^i_{r_1} \right) \cdot \frac{\lambda \omega^j r^j_{r_1}}{v^i_w w^i_{r_1} + b^i g^i_{r_1} + \lambda \omega^j r^j_{r_1}} > 0 \]

**Equation (42) (Federal government problem in the case of labor mobility)**

The first order condition for \( T_L \), derived from the federal government’s maximization problem (40), is:

\[
\begin{align*}
    v^i_w \left[ w^i_{r_1} - 1 \right] &+ w^i_{r_1} \left( \frac{\partial g^i_{r_1}}{\partial T_L} + \frac{\partial g^i_{r_1}}{\partial \tau_L} + \frac{\partial g^i_{r_1}}{\partial \lambda_L} + \frac{\partial g^i_{r_1}}{\partial \omega_L} \right) + b^i g^i_{r_1} \left( \frac{\partial g^i_{r_1}}{\partial T_L} + \frac{\partial g^i_{r_1}}{\partial \tau_L} + \frac{\partial g^i_{r_1}}{\partial \lambda_L} + \frac{\partial g^i_{r_1}}{\partial \omega_L} \right) \\
    + B \left[ G^i_{\ell, T} \frac{\partial g^i_{r_1}}{\partial T_L} + G^i_{\ell, \tau} \frac{\partial g^i_{r_1}}{\partial \tau_L} + G^i_{\ell, \lambda} \frac{\partial g^i_{r_1}}{\partial \lambda_L} + G^i_{\ell, \omega} \frac{\partial g^i_{r_1}}{\partial \omega_L} \right] + G^i_{r_1} \left( \frac{\partial g^i_{r_1}}{\partial T_L} + \frac{\partial g^i_{r_1}}{\partial \tau_L} + \frac{\partial g^i_{r_1}}{\partial \lambda_L} + \frac{\partial g^i_{r_1}}{\partial \omega_L} \right) &\equiv 0 \quad (A10)
\end{align*}
\]

We can simplify (A10) by the following two steps. First, from symmetric assumption:

(1-1): since \( v^i_w \left( w^i_{r_1} - 1 \right) + b^i g^i_{r_1} = v^i \left( w^i_{r_1} - 1 \right) + b^i g^i_{r_1} \), we have \( \frac{\partial g^i_{r_1}}{\partial T_L} = 0 \) and \( \frac{\partial g^i_{r_1}}{\partial \lambda_L} = 0 \);

(1-2): \( \frac{\partial T_L}{\partial \lambda_L} = \frac{\partial T_L}{\partial \lambda_L} \);

(1-3): \( G^i_{r_1} = G^i_{r_1} \) where \( G^i_{r_1} \equiv n' T_L \left( w^i_{r_1} - 1 \right) \)

Second, allowing for labor mobility, we define that \( G^i_{r_1} \equiv G^i_{r_1} + G^j_{r_1} \) and note that

\[ G_{r_1} \equiv 2 \left( G^i_{r_1} + n' \right) \]
Thus, we can rewrite (A10) as:

\[
\left\{ v_w \left( w_{t_i} - 1 \right) + b_i g_i^{j} + B' g_i^{j} \left( \frac{\partial v_i}{\partial t_i} + \frac{\partial v_i}{\partial t_i} + \frac{\partial v_i}{\partial t_i} + \frac{\partial v_i}{\partial t_i} \right) + n' I(2B' - b') \right\} = 0
\]

(A11)

The first order conditions for \( S^i \) and \( S^j \) are:

\[
S^i: \quad v_w \left( w_{t_i} - 1 \right) \frac{\partial v_i}{\partial t_i} + w_n \left( \frac{\partial v_i}{\partial t_i} + \frac{\partial v_i}{\partial t_i} + \frac{\partial v_i}{\partial t_i} + \frac{\partial v_i}{\partial t_i} \right) + b_i g_i^{j} + g_i^{j} \left( \frac{\partial v_i}{\partial t_i} + \frac{\partial v_i}{\partial t_i} + \frac{\partial v_i}{\partial t_i} + \frac{\partial v_i}{\partial t_i} \right) = 0
\]

(A12)

\[
S^j: \quad v_w \left( w_{t_i} - 1 \right) \frac{\partial v_i}{\partial t_i} + w_n \left( \frac{\partial v_i}{\partial t_i} + \frac{\partial v_i}{\partial t_i} + \frac{\partial v_i}{\partial t_i} + \frac{\partial v_i}{\partial t_i} \right) + b_i g_i^{j} + g_i^{j} \left( \frac{\partial v_i}{\partial t_i} + \frac{\partial v_i}{\partial t_i} + \frac{\partial v_i}{\partial t_i} + \frac{\partial v_i}{\partial t_i} \right) = 0
\]

(A13)

Since \( g_s^{j} = 1 \) and \( G_s^{j} = G_s^{j} = -1 \), we can rewrite above two equations,

\[
S^i: \quad v_w \left( w_{t_i} - 1 \right) \frac{\partial v_i}{\partial t_i} + w_n \left( \frac{\partial v_i}{\partial t_i} + \frac{\partial v_i}{\partial t_i} + \frac{\partial v_i}{\partial t_i} + \frac{\partial v_i}{\partial t_i} \right) + b_i g_i^{j} + g_i^{j} \left( \frac{\partial v_i}{\partial t_i} + \frac{\partial v_i}{\partial t_i} + \frac{\partial v_i}{\partial t_i} + \frac{\partial v_i}{\partial t_i} \right) = B' - b' \quad (A12')
\]

\[
S^j: \quad v_w \left( w_{t_i} - 1 \right) \frac{\partial v_i}{\partial t_i} + w_n \left( \frac{\partial v_i}{\partial t_i} + \frac{\partial v_i}{\partial t_i} + \frac{\partial v_i}{\partial t_i} + \frac{\partial v_i}{\partial t_i} \right) + b_i g_i^{j} + g_i^{j} \left( \frac{\partial v_i}{\partial t_i} + \frac{\partial v_i}{\partial t_i} + \frac{\partial v_i}{\partial t_i} + \frac{\partial v_i}{\partial t_i} \right) = B' \quad (A13')
\]

We substitute (A12') and (A13') into \( (B' - b' + B') \) in the right hand side of (A11), respectively.

Thus

\[
2B' - b' = v_w \left[ n_w' \left( w_{t_i} - 1 \right) \frac{\partial v_i}{\partial t_i} + w_n' K \right] + b_i g_i^{j} \left( \frac{\partial v_i}{\partial t_i} + \frac{\partial v_i}{\partial t_i} + \frac{\partial v_i}{\partial t_i} + \frac{\partial v_i}{\partial t_i} \right) + g_i^{j} K
\]

\[
+ B' \left[ G_i^{j} \frac{\partial v_i}{\partial t_i} + G_i^{j} \frac{\partial v_i}{\partial t_i} + G_i^{j} \frac{\partial v_i}{\partial t_i} + G_i^{j} \frac{\partial v_i}{\partial t_i} \right] + G_i^{j} K + G_i^{j} M
\]

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where \( K \equiv \left( \frac{\partial \omega}{\partial \alpha} + \frac{\partial \alpha}{\partial \omega} \right) + \left( \frac{\partial \alpha}{\partial \beta} + \frac{\partial \beta}{\partial \alpha} \right) + \left( \frac{\partial \beta}{\partial \gamma} + \frac{\partial \gamma}{\partial \beta} \right), \)

\[
M \equiv \frac{\partial \omega}{\partial \alpha} + \frac{\partial \alpha}{\partial \omega} + \frac{\partial \alpha}{\partial \beta} + \frac{\partial \beta}{\partial \alpha} + \frac{\partial \beta}{\partial \gamma} + \frac{\partial \gamma}{\partial \beta} \].

Because both states are symmetry (that is \( \frac{\partial \alpha}{\partial \omega} = \frac{\partial \omega}{\partial \alpha} \)) and \( \frac{\partial \alpha}{\partial \omega} + \frac{\partial \omega}{\partial \alpha} = 0 \), \( \frac{\partial \alpha}{\partial \beta} + \frac{\partial \beta}{\partial \alpha} = 0 \) and \( \frac{\partial \beta}{\partial \gamma} + \frac{\partial \gamma}{\partial \beta} = 0 \), respectively.

Moreover, since \( \frac{\partial \omega}{\partial \alpha} = 0 \), \( K = M = 0 \). And also \( G_i \left( \frac{\partial \alpha}{\partial \omega} + \frac{\partial \omega}{\partial \alpha} \right) = G_i \left( \frac{\partial \alpha}{\partial \beta} + \frac{\partial \beta}{\partial \alpha} \right) \). Thus, we can rewrite;

\[
2B' - b'_j = v'_w \left( w'_t - 1 \right) \left( \frac{\partial \omega}{\partial \alpha} + \frac{\partial \alpha}{\partial \omega} \right) + b'_j \left( \frac{\partial \alpha}{\partial \beta} + \frac{\partial \beta}{\partial \alpha} \right) + B' \left( \frac{\partial \beta}{\partial \omega} + \frac{\partial \omega}{\partial \beta} \right) = 0
\]

(A14)

Substituting the (A14) into (A11), we obtain;

\[
v'_w \left( w'_t - 1 \right) + b'_j \left( \frac{\partial \Gamma}{\partial \omega} + \frac{\partial \omega}{\partial \Gamma} \right) + B' \left( \frac{\partial \alpha}{\partial \omega} + \frac{\partial \omega}{\partial \alpha} \right) = 0
\]

(A15)

**Proposition 3 (Derivation of the second best outcome with labor mobility)**

From (A15) and \( v'_w \left( w'_t - 1 \right) + b'_j \left( \frac{\partial \alpha}{\partial \omega} + \frac{\partial \omega}{\partial \alpha} \right) = \frac{\lambda \omega' r'_i \Delta}{v'_w w'_t + b'_j \left( \frac{\partial \alpha}{\partial \omega} + \frac{\partial \omega}{\partial \alpha} \right)} \), the federal government has to satisfy

\[
G_{B, NB, mobile}^r = \left( \frac{\Delta}{v'_w w'_t + b'_j \left( \frac{\partial \alpha}{\partial \omega} + \frac{\partial \omega}{\partial \alpha} \right)} \right) \left( \frac{\lambda \omega' r'_i}{B'} \right)
\]

satisfy

\[
\lambda \omega' r'_i = \left\{ B \left( v'_w w'_t + b'_j \left( \frac{\partial \alpha}{\partial \omega} + \frac{\partial \omega}{\partial \alpha} \right) \right) G_{B, NB, mobile}^r \right\} \Delta r'_t
\]

and \( r'_i \left( 1 - w'_t \right) f' h^2 \Delta \) into state government optimal condition (40), and by using \( b'_j = 2B' \), we obtain:

\[
\frac{n'2B'}{u'_t} = \frac{1}{1 - \frac{r'_t}{r'_w} - f' h' \Delta + G_{B, NB, mobile}^r} \left[ 1 + \left\{ \frac{n'2B'}{u'_t} \right\} \left( G_{B, NB, mobile}^r \left( w'_t - 1 \right) f' h \Delta \right) \right]
\]

This expression is just same in (A6). Thus, we get Proposition 3.
REFERENCES


